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Habit Formation, Demand and Growth through product innovation

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Abstract

Growth theory has mainly focused on process innovation, either through an increase in quality of the product or a reduction on the cost. The main contributions in growth theory that includes product innovation has been done in the Dixit and Stiglitz framework. This framework works with oversimplifying restrictions on the demand side: Preferences of consumers are assumed to be constant and equal for all goods.
This paper introduces vertical and horizontal differentiation in final goods. Goods are different in their habit formation parameters. Innovation is not the normal reduction in costs but an increase in the capacity to satisfy consumers' needs. Growth in this model is defined as the growth of the final value added.

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1 Introduction

In this chapter we shall present an economic growth model. The idea is to highlight one effect of technology which has often been neglected when discussing technical change. It is well known that technological change reduces production costs, however technology also reshapes society's consumption habits and customs. Successful inventions have an effect on consumer choice, offering new opportunities. A continuous flow of successful inventions affects the wealth generated in the society as they develop into habits. In our model we shall study one specific direct effect of technology on final demand; the creation of new needs. Our model isolates this effect from process innovation to clearly understand its effects although we know product innovation never happens in isolation. The main motivation is to reflect on the fact that technical change does not just reduce production costs but also generates needs and reshapes preferences in the society.

We believe this effect of technology is clearer in advanced societies, where all basic needs are covered. Every year a mass of innovations comes onto the market, some of which will become habits for a proportion of the population and others will disappear from production. However over the years the number of needs continues to increase. Comparing the options of today's consumer to those a consumer had a hundred years ago, the number and variety has increased. However, not only the absolute number of options has increased, today's consumer also considers more items to be essential than our grandparents did. This is the focus of the chapter: to study how this increase in needs affects economic growth.

In real life, technical change through introduction of final good inventions is very chaotic with many innovations generated at the same time, strongly affected by continuous natural selection. Some of the year's many inventions will become habits, others will disappear. We shall concentrate on the innovations that become consumer habits; these we consider are the real innovations. In reality there is no relation between the habits generated by one years innovations, and the habits created by the next. However we assume that in each period a single innovation will be introduced and this innovation will create a habit stronger than earlier ones. In the previous article preference evolution was introduced (see Garcia-Torres (2009)). It was argued that to understand preference evolution two key factors need to be taken in consideration: habit formation and novelty. Both are relevant, however in this article the main attention will be on habit formation. From the conclusions of the previous chapter, we know that a certain amount of novelty has to be perceived by the consumer at the beginning of the life of a product. In this article the assumption is that all new goods reach this level. We shall discuss the macroeconomic effects of product innovation, concentrating on habit formation and economic growth.

Consumer behaviour: evolution of preferences and the search for novelty

In this article when we talk about growth we are not referring to physical growth of the output. Our point is that, given a more active role of the demand, it is possible to have growth of added value and utility without increasing the amount of physical objects produced. We will work with a single production input: labour. In equilibria the total amount of labour is fixed and all industries have the same constant production coefficient. In this sense the total final quantity of goods produced by the economy will be constant. The variety of final goods is constantly changing, even after equilibria is reached, and so the total value of the production will be constantly increasing. New goods are superior to previous ones in their capacity to form habits and also have higher income elasticity than old goods. In the equilibrium the total value of the production will be increasing, while the physical amount produced will be constant. This growth is possible because each time a new good arrives the utility of individuals is increased.

Behind most economic growth theories is the assumption that what matters most is the total physical production. Economists have attempted to discover the reasons that explain increase in physical output, either modelling ways to increase factor accumulation or providing reasons that explain reduction in production coefficients. In both cases it is a matter of producing more; in this sense we argue that most economic growth theories are biased towards supply. The dynamics of the demand in these theories are extremely simple, assumed in most cases to be equal to supply and that all production is bought.

Papers that study how technical change affects demand and growth can be divided into two groups, those in which new products bear a horizontal relationship to existing goods and those with a vertical relationship:

- In the case of a horizontal relationship technology increases the variety of products, all of which contribute equally in every moment of time to the general utility.
- When talking of a vertical relationship we mean the quality/utility of each existing good is increased by technology.

Our model combines both ideas. In each period we have both horizontal and vertical relationships to existing goods because in the period we introduce a new good of superior utility. Our interpretation of increase in utility is that the habit or need of the new product is superior to that of earlier ones.

In the third chapter of their book, Grossman and Helpman (1991), present a variety of models where technology affects the demand, increasing the variety of goods and therefore growth. We discuss the first two presented there since our model can be compared with both. In the first model, presented in the first section of the book, they treat technology as an entirely private product where entrepreneurs invest resources in order to develop unique goods. An ordinary production function relates input (primary factors of production) to output (blueprints for new goods). Product designs are assumed to be proprietary information, either because their details can be kept secret or because patents effectively deter unauthorized uses. Each new product substitutes imperfectly for existing brands, and innovators exploit limited monopoly power in the product market. The assumption is that the potential for developing new products is unlimited; however in this model growth ultimately ceases. Even if the resource cost of creating new goods does not rise, the economic return on invention may decline as the number of products increases. In the model presented in the next section of the quoted chapter, they modify the innovation process to treat a part of the innovation as knowledge capital, which is considered a public good. This modification allows them to present a self-sustaining process of endogenous innovation. We work within a similar framework, with the distinction that each good gives a higher utility on the demand side and a higher profit is associated to it on the supply side. Therefore our profit rate for each future good is constantly increased, making retaining the R&D sector and investing in it worthwhile. Our specification allows us also to present a self-sustaining process of endogenous growth, not by introducing knowledge capital as a public good but by introducing a more relevant role of the demand. Since each new variety has a higher utility it also has associated higher profits that do not reduce the economic return of the final good sector.

The article is structured as follows: in section 2 we discuss the state of the issue and review the literature, in section 3 we explain the reasons and ways in which demand affects growth, section 4 formalizes the discussion presenting the different agents of our model and section 5 discusses the equilibrium; the chapter's last section draws a conclusion.

2 Demand and growth in the economic literature

The industrial revolution made one idea clear: machines and capital are able to produce more with less people. It is a supply-driven effect. The main ideas behind the recognized growth theories, especially after Solow (1956), are biased toward the supply side. The theories concentrate on factor accumulation and productivity and the conditions that can increase them. For process innovation anything that increases productivity through a reduction of costs or an increase in the quality of production factors will be translated into higher growth rates. The mechanism causing this to happen varies with the model. We shall briefly review the main sources of growth for each of them:

- Solow (1956) assumes that more savings are transformed into more investment. Investment basically means more capital and more machinery in the production process. The notion is that with more capital transformed into machinery the same people can produce more. Capital presents decreasing returns to scale, therefore in the steady state growth is determined by two main sources, the growth of technology and the growth of the population.

The strongest criticism made of this model was that the factors explaining growth were outside the model. As a reaction the endogenous growth literature was born in which each model gives a different reason to explain increased production:

- Arrow (1962) presents the idea of knowledge spillover. Knowledge generated can be used in many different places without increasing the cost, therefore it will increase total output.

- Lucas (1988) introduces the argument of human capital. It is not just a question of more machinery or more spillovers, but the fact that more highly qualified people will be able to produce better and more. He presents two main methods for generation of human capital: schooling and practical experience.

- Romer (1990) reflects on the increasing complexity of production processes. He argues that innovation affects the number of inputs used during production so that we are able to produce more. Thus innovation generates more inputs, which in turn enhances productivity.

- Aghion and Howitt (1992) introduce the Schumpeterian concept of creative destruction in their model: a new production process makes its predecessors obsolete. In a way it is not the increasing number of goods but better production processes which generates growth.

The role of demand in these theories is very simple; it is always made to equal the supply. The dynamics of demand are neglected and it is left to passively absorb all production. All the cases previously discussed assumed homothetic preferences. It is interesting to note that demand was much more important in the economic discussion before Solow wrote his article. Previous theories were much more concerned with the interaction of supply and demand, allowing for different dynamics in both sides. For example, Harrod (1939) presented a model of growth in which the key driver of the growth process is the level of the community's income. Income dynamics determine the supply and demand of savings. And then in Domar (1946) we find a rich discussion about the determinants of the growth of production (increase in the working population, labour productivity...) together with a presentation of different determinants for the growth of demand (increase in the national income...). In addition he analysed cases in which the rate of growth of each determinant is different. However the only things remaining in the text books from these earlier ideas is that they use a Leontieff production function, a criticism made by Solow (1956). Only some ideas introduced by Ramsey (1928), when he was looking for a theory of savings, seem to remain from the exogenous growth model. In his article, savings are not a constant proportion of production but depend on decisions made by consumers.

Recently some new growth theories have taken an interest in the role of demand. Three main interrelated branches can be distinguished in the literature: the causality from growth and savings, the relevance of habit formation, and the importance of different new products. As we have commented earlier in the section, the majority of growth theories take the view that production is too low. Anything increasing production is assumed to be good. More production means more income, and more income generates increased savings that are assumed to be transformed into investment. This higher investment increases productivity and we start a new loop, convinced that more savings generate more growth. One possible explanation for this conviction is that the dynamics of demand are never acknowledged by any of the theories.

If demand drives the growth process, then a reduction in savings, which implies an increase in consumption, will generate more growth and consequently more savings. But the direction of causality is changed, no longer leading from more savings to growth, but from more growth to savings. Carroll and Weil (1994) present some empirical evidence about this. They use a sub-sample of 68 countries from the Summer and Heston data base, making a cross-sectional analysis, and bring empirical evidence to prove that periods of high incomes are followed by periods of high savings. They also exploit American household data from the Panel Study of Income Dynamics (PSID) and other sources to investigate whether expectations of growing incomes will generate higher saving rates - they find some positive evidence. Having studied both macro and micro levels they conclude that causality goes from growth to savings. In Carroll et al. (2000), we find a theoretical model supporting the empirical evidence. Consumers maximize in a Ramsey framework a Constant Relative Risk Aversion (CRRA) utility function which has been modified to introduce habits. The model is based on the presence of habit formation in consumption patterns and they show that in a standard endogenous growth framework the presence of habits is sufficient to show growth-to-savings causality.[1] They study how changes in habit formation persistence might affect the steady state growth. Falkinger and Zweimuller (1997) present a different point of view which is more related to Engel's laws: higher incomes give a greater possibility to spend more on luxury goods than low incomes. They present some empirical evidence

[1]Even though in most growth models savings and total output normally move in the same direction, they study the variation of the saving function with respect to the growth rate. The sign of the first derivative of the saving function with respect to growth rate can be changed by the presence of habits.

based on data provided from the United Nations Comparison Project (ICP). From their study of 27 countries they drew two main conclusions, stating inequality affects growth negatively. The reason given is that high income inequalities imply that less people can afford luxury goods, which therefore has a negative effect on growth. They also test the hypothesis that increasing the number of goods has a positive impact on productivity levels and reject it. Attanasio et al. (2000), using data from the World Bank, attempts to study Granger causality between the three relevant variables: growth, investment and savings. Granger causality studies how a variable (the one caused) correlates with the lagged values of the others. The most robust result is a negative relation from investments to growth.

Demand is modelled in the growth literature in such a way that goods are preferred in a proportional manner. An increase in consumer income proportionally affects the total consumption of each good. These are so-called homothetic preferences. As soon as this assumption is disregarded, demand plays a much more determinant role in growth models. One interesting result is presented by Echevarria (1997). She is working with non-homothetic preferences and sectoral change. She presents a general Solow model with three sectors, agriculture, manufacture and services. Each sector has a different exogenous rate of technical change, therefore the productivities are different. Depending on the phase of development of the country, the sectoral composition moves from agricultural to service sector. Because preferences are not homothetic the composition of the demand affects the growth rate of the country. While she focused on differences in international growth rates, Kongsamut et al. (2001) in a similar theoretical work based on sectoral change, are able to reconcile Kaldor stylized facts with the massive reallocation of labour from agriculture and manufacture to the service sector. They show that a balanced growth path exists only under complex and restrictive knife-edge parameters of both technology and preferences. Similarly Felice and Bonatti (2004) present a model with two sectors: a stagnant service sector and a manufacturing sector with AK production function. Working with non-homothetic preferences they analyse different patterns of demand for each sector. The role of demand, i.e. the proportion of expenditure devoted by individuals to the manufacturing and stagnant sectors, becomes crucial in determining whether the economy displays perpetual growth or stagnancy.

One of the possible reasons for reversing the causality from growth to savings is the presence of non-separable time preferences or, in other words, habit formation processes. The idea was first presented by Duesenberry (1949). In his book he reflects that past consumption shapes today's preferences. The first formal studies of habit formation and growth come from Pollak (1970) and Ryder and Heal (1973). They focused their efforts on explaining the relations between past consumption and preference formation at the individual level. In Osborn (1988) we find some empirical evidence about the relevance of habit formation. He uses consumption data for durables in the UK. In an attempt to explain consumer behaviour, he rejects the hypothesis of the life cycle. He concludes that habit formation with seasonal adjustment is the best way to explain the consumer's patterns.

When working with preference formation the notion of conspicuous consumption plays a prominent point. Van de Stadt et al. (1985) work with two forms of annual data about Dutch households. They incorporate two ideas, the idea that habit formation is relevant and that individual consumption depends on the average consumption level of the society. As an approximation they find that the consumption level of the society explains 1/3 of the consumption pattern, the remaining 2/3 being explained by individual past habit formations. A microeconomic model that could explain this pattern was presented in section **??**. One of their main conclusions is that habit formation is too important to be neglected in cross-sectional analysis. Some research has been done in this area on the important criteria when modelling non-separable time preferences. Three options are observed in the literature: a) inward-looking preferences, in which case the consumer is affected only by her own past consumption (habit formation), b) the consumer behaviour is determined by society's consumption level (also called "catching up with the Joneses") or c) a mixture of the two. We shall focus for this review of paper only on the first and third options; the second one has a strong connection with conspicuous consumption literature and is therefore outside the scope of this chapter. Carroll et al. (1997) study two separate models: one concerned only with inward-looking habit formation, and the other based only on societal habit formation. They study how negative shocks to capital, savings and growth affect the transition to the steady state under the two assumptions about preference formation. They conclude that the decline in growth rate will be less if habit formation is determined by outward-looking preferences. Carroll et al. (2000) focus on the intensity of habit formation, comparing weak habits to strong ones. The higher the persistence of habits, the higher the impact on growth and on reversing the causality from growth to saving. The preceding two models are endogenous growth models with the simplest AK production function. Alvarez-Cuadrado et al. (2004), present a similar model but introduce a production function that presents decreasing returns to scale on capital. This is an interesting way of modelling preference evolution wherein habit formation is a geometric mean of individual past consumption and the consumption level of the society. They compare three cases: normal time-separable preferences, catching up with the Joneses and habit formation. Their findings concur with the rest of the literature, that non-time-separable preferences affect the dynamics of growth. When catching up with the Joneses, the differences in growth arise from a consumption externality, in the case of inward-looking habit formation they arise from the fact that individuals smooth consumption. Alonso-Carrera et al. (2004) analyse a model with a standard neoclassical production function exhibiting constant returns to scale. The equilibrium exhibits transitional dynamics driven by both habit formation and decreasing returns on capital. They show that to see some effect of societal consumption level on growth, it has to affect habit formation dynamics. They find two main ways to treat this effect: either by modelling society's average consumption as a consequence of its past consumption level [2] or by affecting the habit formation process of each individual.

In the previous paragraph consumption was taken at the individual level; there was no distinction at product level. The role of new goods and their effects on growth was first studied by Grossman and Helpman (1991) in the third chapter of their book, as we have already commented in the introduction. They used the framework generated by Judd (1985), which was a study of patent length versus the number of goods. In their model using Dixit and Stiglitz (1977) monopolistic competition, they determined the optimal number of different goods for a society. Two opposing driving forces permit equilibrium. On the one hand consumers love variety, on the other hand firms face monopolistic competition, so the higher the number of new goods, the lower individual profits become.

However in their model goods are different but symmetric. Engel's laws provide one method of removing the symmetry so as to understand the importance of having different products. New products are assumed to be more expensive, therefore when they are introduced only higher-income consumers can afford them. If these new products are a source of growth, a more unequal distribution of incomes has a negative effect on growth. Goods follow a hierarchy from basic needs to luxury goods. Zweimuller (2000) set up a theoretical model collecting some of these concepts. In a framework similar to Grossman and Helpman (1991), consumers differ in their initial assets. Goods are demanded hierarchically, with new goods showing higher income elasticity. He shows that inequality affects growth because it discourages innovation. If relatively few consumers have high income, then demand for the new good is expected to be low, and so innovators are discouraged because they can not get back the sunk costs of innovation. Foellmi and Zweimuller (2002) present in a similar framework a model in which Kaldor facts and sectoral production and employment changes are reconciled. Goods are preferred in a hierarchical way; as the economy grows there is a shift of labour from necessities to luxury goods. In equilibrium the industries taking on more labour coexist with those shedding it; nonetheless macroeconomic aggregates grow at a constant rate. Foellmi and Zweimuller (2004) analysing the monopolistic competition model, come to the conclusion that, if preferences are not homothetic, then income distribution will always affect growth through markups and diversity of goods.

[2] In a way this is habit formation for the entire society. The whole society is treated as a single individual.

A different argument is presented by Aoki and Yoshikawa (2002). They reflect that in neoclassical growth theories the main constraint is diminishing returns on capital and add the argument that demand saturation is also determinant. Each good presents an S-shaped diffusion process at different states; they conclude that demand saturation constrains steady state growth. Greenwood and Uysal (2004) introduce the argument of demand on new goods in a sectoral change model. They present three sectors: agriculture, a capital-producing sector and a "new goods" sector, each growing at a different exogenous rate. They conclude that as the economy grows and incomes rise consumers demand new goods because of a love of variety. A negative consequence of this effect is that the relevance of the agriculture sectors always decreases. Benhabib and Bisin (2000) present an interesting point: the interaction of monopoly power with marketing strategies can create negative welfare effects. Also, following a Grossman and Helpman (1991) framework, firms do marketing. Marketing expenditure affects preference formation through income elasticities. They present different ways in which preferences are affected by marketing strategies. They studied how the Dixit and Stiglitz (1977) monopolistic competence could be affected by marketing power. Similarly, Ravn et al. (2004) focused on the effect of habit formation on different products; they called it the "Deep Habit" formation process. They argue that in a habit formation framework firms may adjust their pricing strategies to habits, so giving rise to the countercyclical mark-up process empirically evidenced.

The main difference in our model from the rest of the literature is the approach toward technical change. In our model there is a flow of innovations, each innovation being superior to the previous one in its capacity to satisfy the consumer. Technical change for us means only product innovation; process innovation does not exist so production coefficients are always constant. In this way it differs from models dealing with sectoral change (Kongsamut et al. (2001); Echevarria (1997); Felice and Bonatti (2004); Greenwood and Uysal (2004)). In these models it is always assumed that sectors differ in the intensity with which technology affects production coefficients. We study the presence of habit formation at the product level, thus differing from Carroll et al. (1997, 2000); Alvarez-Cuadrado et al. (2004); Alonso-Carrera et al. (2004), who treat consumption at aggregate level. The flow of innovations is such that all goods are basic necessities (income elasticity is always positive and equal to one), so modelling innovation to generate needs not luxury goods (in opposition to Zweimuller (2000); Foellmi and Zweimuller (2002, 2004)). We have no saturation level for any of the goods; in this we differ from Aoki and Yoshikawa (2002). Despite the fact that marketing has an important role in the introduction of innovations, the success of the innovation depends on its own capacity to satisfy the consumer; we do not allow firms to affect preferences through marketing investments. In this aspect our work differs from Benhabib and Bisin (2000). Ravn et al. (2004) study habit formation in conditions where decisions are maximized by both consumers and firms. We have simplified their framework to assume that neither firms nor consumers have the capacity to do this. Ravn et al. also considered that habits are under conscious control but our assumption is that habits develop at the subconscious level as was explained in article **??**. Our main motivation is to prove that, in a model excluding process innovation but in which habits are considered, product innovation can increase the marginal propensity to consume in the economy.

3 The engine of growth: an infinite number of needs

To introduce the motivation of the chapter, let us start by quoting Galbraith (1958) and his formulation of dependence effect:

> "As the society becomes increasingly affluent, wants are increasingly created by the process by which they are satisfied. Increases in consumption, the counterpart of increases in production, act by suggestion or emulation to create wants."
>
> *"The Affluent Society" p.135*

Although Galbraith was talking about economic change in general, we would like to apply his thoughts to technological change in particular. We shall argue that nowadays, when technology plays such a prominent role in advanced societies, a part of technological change is concentrated on the generation of new needs. In this article we present a model in which modern societies are able to grow by creation of new wants.

In models such as that presented by Zweimuller (2000), Foellmi and Zweimuller (2002, 2004) the main argument is that new products are always luxury goods and only part of the society can afford them. Our view of the process is slightly different in the sense that we believe the real impact of an innovation occurs when it becomes a necessity for the whole society. Therefore in our model an innovation is considered to be such only if it is a necessity for the whole population. The products considered here will be neither luxury nor basic goods; the income elasticity for all the goods considered will be strictly positive and equal to one. We feel product innovation has its biggest impact on society when innovations are used and needed by the majority of the population.

The main motivation is our belief that technological change is generating habits in our lives and that, if we study from a historical point of view the life of a representative consumer in a developed country over the last hundred years, we observe that the number of things bought has increased over the years. Before going into the details of model let me present some examples in which we can see needs being generated. The phenomenon is not a rapid process, but a very slow and subtle one and therefore hard to recognize. We would like to reflect on one example comparing intergenerational consumption patterns.

In this comparison we highlight one process: how an innovation becomes a necessity. We are not talking of Engel's laws, by which an increase in income produces a greater increase in luxury goods (the relationship of this state of affairs and growth has been analysed by Falkinger and Zweimuller (1997); Foellmi and Zweimuller (2002, 2004)). We focus on the fact that what was a luxury innovation for our grandparents is for us a need. Your grandmother as a child probably considered a refrigerator to be a luxury. Nowadays it is a necessity found in the majority of homes. Now supposing we ask if you need a new gadget, which uses biotechnology and could make your life easier, your answer could be yes or no. But if we ask whether you could live without this gadget, the answer will be affirmative. If we went back in time to when the refrigerator was invented and asked our grandparents whether they could live without a refrigerator, the answer would also be affirmative. Nowadays the majority of people would reply to the same question in the negative. But why this change in the answer within two generations? Habit formation. We are so accustomed to refrigerators that we can no longer live without them. What was a luxury innovation for our grandparents has become a need for the world.

The refrigerator is just one example of many other goods and services that are constantly becoming considered more necessary and less luxurious. This evolution is the effect of technical change when it is observed from the demand side: an increase in goods and services that are being used by people. Nobody would argue that a refrigerator is a basic human need such as food or shelter. In this sense the refrigerator is not essential for human life. If a developed society by some means reached a crisis in which refrigerators could no longer be bought, the general utility of such a society would decrease but it would not directly effect the survival of the society. The new situation of the society will be considered as underdeveloped when compared with the earlier one because we associate economic development with more and more things being available to consumers. As time passes and economic development continues, some of these goods become standard needs for people. Thus in the absence of crises economic evolution implies an increase in consumer habits, which in turn implies an increase in the general utility of the society. During this process the things being needed and produced increase in value and so the wealth of the economy is constantly being raised.

Innovations are classified in the technical change literature as incremental or radical. Radical innovations are ones that change the entire production process, while incremental ones make only minor changes within a technological paradigm. This classification shows the supply-side bias of the technical change literature. We would argue that a radical innovation is one that in the long run can become a habit for a large proportion of the population. An incremental one would be important for a smaller proportion or for a short period of time. In this article we will mainly focus on this type of radical innovations.

Considering the flow of innovations, each innovation is superior to the previous one as it has a higher capacity to generate consumer utility. This will allow monopolies to earn higher profits on the latest innovations. Profits will be related to earlier ones in a constant fixed relation. In the real world, innovations with higher capacity to increase utility arrive in a haphazard way. In the model we have assumed that they arrive in order so as to derive an analytical solution. A less orderly manner could be studied using individual-based simulations, however this is left for further research.

Our model fits into the endogenous growth literature, which has been strongly criticized as being unrealistic. Jones (1995a,b), working with some empirically criticized R&D-driven models, argues that population is growing in most countries of the world and that some models under these circumstances produce explosive growth. His criticism could also be applicable to the model we present, however our concern is not to produce a realistic description of the world but to model a phenomenon which has not been studied by many people. Theoretical modelling is used to prove that under some assumptions an increase in the needs of the population, coming through technical change, generates growth. The population is always willing to buy more; the means by which this happens will be explained in the next sections. However we emphasise that R&D is aimed at new goods, not to develop new production processes.

With no process innovation and a constant amount of inputs, the total produced amount in equilibrium will always be the same. The source of growth comes from introducing new varieties with increasing utility. Demand, because it reaches higher utility values will be willing to pay more. Therefore even though the total produced amount is constant in equilibria, the value of the production keeps on rising. To clarify this point we first give an example of how this happens and then see how it differs from normal inflation. Imagine a country where all chairs are painted in a single colour, either red or white. The price of each chair is one unit. A designer imagines red and white striped chairs. She patents the idea, produces the good, and decides to sell the new striped model at the price of two units. The cost of the input is the same. For every unit she is able to sell there will be an increase in the value of the sector's sales. A new idea, in this case a new manner of painting chairs, has generated a new good for which some consumers are prepared to pay a higher price. The value given by demand is higher thereby generating growth. This is how growth occurs; there is an increase in added value but not in the quantity produced, which remains constant. In our model the variety of the goods is constantly increasing, each of them with a stronger habit; and it is this that produces the price rise and increased value of the production - a combination of the consumer "love of variety" and a "superior utility" of each new variety.

One could argue that this is just inflation, but in the model we are introducing unrelated new goods. Inflation is defined as the increase in prices of a constant basket of goods. In our model the basket of goods will be changing so the rising prices cannot be described as inflation. The model's higher prices are a result of the overall rise in the level of society's utility. Therefore we have an increase in the total nominal value and an increase in individuals' well-being.

4 The mathematical model

In this section we present a model to show it is possible to have growth both in the utility function and in the nominal value of the production even with constant inputs and without cost reductions. Growth comes through the introduction of goods that are increasingly sought after by the population. People are willing to pay more for the latest innovations since they know they will experience higher utility. As well as the number of goods and the total added value produced by the economy, the utility experienced by the population always grows. First we shall briefly introduce all the different parts of the model, then describe in more detail the decisions taken by agents in each step before ending with the general equilibrium.

Technological change is represented in this model by product innovation; the arrival of a new innovation will determine a new period. There are two types of agent: individuals, who maximize utility, and firms, which produce in two sectors: final goods and R&D. The distribution of people between these two sectors will be an exogenous decision taken by the economy; in both sectors it will be held constant. The amount of labour devoted to R&D will determine how much of a habit (increase in the utility) each new good becomes.

- Individuals
  - Individuals maximize their utility function deciding on:
    - How much of each product to consume (x_1, x_2....),
    - How much to save (s_t).
  - Each individual inelastically supplies one unit of labour every period.
- Firms

  Using labour as the only input will work on two sectors:

  - A production sector of final goods,
  - An R&D sector producing blueprints.

These agents interact in four different markets:

- A final goods market where each product is sold at price p_{it}.
- A blueprint market.
- A labour market with a homogeneous price w_t.
- An equity market that is regulated by the interest rate r_t

We now present the decisions that are made by each of these agents in each market in the short run. Later we shall give a definition of the long run equilibrium.

4.1 Time and the arrival of new products

The arrival of innovations is a deterministic process. The arrival of a new innovation defines the beginning of a new period; only one innovation is possible per period. The method of modelling technological change is similar to that of Aghion and Howitt (1992), but has no uncertainty and is applied to product innovation rather than process innovation.

In the previous article it has been extensively discussed how people develop habits. We mentioned several options and several possible outcomes (see Garcia-Torres (2009)). Once an innovation appears there is an adjustment time which allows consumers to decide on an optimal quantity to buy based on prices and habits. As was explained, this is a long process. In the previous article it was also shown that sooner or later the consumer will reach a stable solution if no external shocks appear.

Figure 1 defines two terms which we shall use later when introducing further assumptions. A 'period' begins when an innovation arrives. The time span from this arrival to the arrival of the next innovation we shall define as the time "between periods". Immediately after the new innovation has arrived up to the moment at which the next one appears we shall designate as time "within a period". As an analogy we can imagine new innovations arrive at the start of each year (between periods) although the consumer makes decisions on a daily base (within a period). The decisions taken by the consumer during 'within the period', assuming a large number of (possibly daily) subperiods, will assure us that the consumer has developed habits for existing goods and that she knows exactly how to distribute her income among the existing goods. The proportions at the end of the time within the period will be constant for each existing good. Thus at the end of the year, due to daily decisions, the consumer will have stabilised the distribution of her income, and be buying a constant proportion of each good, although different for each of them. It is important to grasp the distinct meaning of these designations of 'between periods' and 'within a period'.

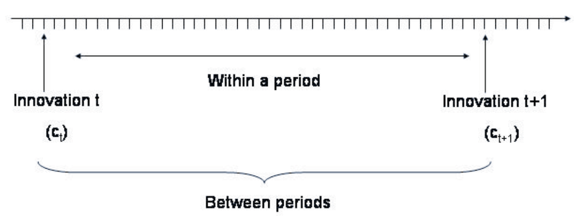

Figure 1: Time evolution between and within periods

The previously stable equilibrium of allocation of resources will be broken by the arrival of the next innovation. In this model we concentrate only on these shocks. In effect we shall be working with a flow of interrelated innovations. New innovations need time to be brought to market, during which the consumer will have repeatedly bought the good (each purchase ending a subperiod) and gradually developed this consumption into a habit. Every time a new innovation arrives the consumer will be in equilibrium, so that we can use the solution presented in the previous article (Equation **??**).

Products differ in their capacity to generate experienced utility in the consumer. The term c_i is the intrinsic capacity that the good has to produce experienced utility. In this model innovation is defined as the arrival of a new good which has higher c_{i+1} than all the previous c_is present in the market. The procedure is similar to that of Aghion and Howitt (1992). According to them the arrival of a new innovation contributes in a fixed proportion to production cost reduction. In our case it will increase the capacity of the new good to be needed by the population. Our model also differs from theirs in that innovations arrive with certainty. There is a relation between the last c_i and c_{i+1}; every time an innovation arrives it will happen such that:

$$\frac{1}{c_{i+1}-1}=\gamma\frac{1}{c_i-1} \qquad with \quad \gamma>1 \; and \; c_i<1 \; i=1,2,3... \tag{1}$$

This is also the definition of technological change in this model. Solving for c_{i+1} we have:

$$c_{i+1}=\frac{\gamma+c_i-1}{\gamma} \tag{2}$$

γ in this model is not a parameter but variable[3] defining the relation between the last innovation and the previous one. Is defining the jump in the utility and it is related to the total labour devoted to R&D. The solution of the system, will prove that this variable is always in equilibrium larger than one. And if $\gamma > 1$ then:

$$\lim_{i\to\infty} c_{i+1}=1 \tag{3}$$

The arrival of new innovations is a deterministic process. In every time step we have a new innovation. The contribution of the latest innovation compared to the previous one is known, because it is known the amount of labour devoted to R&D in the manner discussed in subsection 4.3.2. All members of society know that innovations are going to arrive so this will affect their savings decisions.

Figure 2 depicts the events happening in every time step. As soon as the patent is sold the monopolist starts producing the new good. [4]

4.2 Individuals

Modelling time and the arrival of innovations in the manner presented in the previous section, and assuming individuals only choose present quantities,

[3]For simplicity we will assume that in principle the system is in equilibrium, i.e. there is a constant distribution of labour between our two sectors. If the system is outside the equilibrium, in each period until the equilibrium is reached we will have a different γ. This case will be analysed after the equilibrium is defined in subsection 5.1. We will afterwards study this case allowing for different γs. That will be section . This is done for simplicity of the algebra.

[4]From section **??**, we know that the consumer needs some time to develop habits. We also know that the novelty associated with that good has to rise in order for the consumer to buy the first unit. Therefore we will assume that, even though the good is produced, in the first period it will produce no profits because all benefits will be used, in the form of marketing expenditure, to raise its novelty. The concept is as follows: the R&D sector comes up with an idea and also markets the product to raise the novelty value to the required minimum level. They will sell the product to generate some profits. To keep things as simple as possible we will assume that the total profit after paying for marketing expenditures is zero. When they start getting monopoly income the product's patent will be bought by a final goods firm and from this time on it is the final goods firm which earns monopoly income.

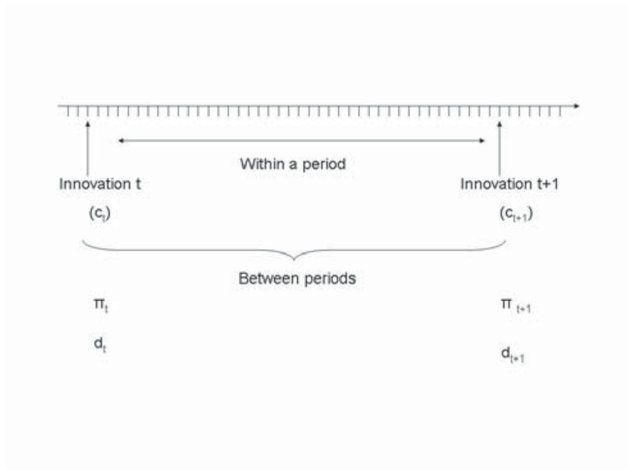

Figure 2: Arrival of innovations

i.e. they do not have control over their habit formation, we can assume intertemporal separability of lifetime utility between periods and apply two stage budgeting. In the first stage we will derive a solution within the period where the budget is constant, and in the second we will treat the dynamic problem of optimal allocation of lifetime expenditure between time periods.

We could allow individuals to have control over their habit parameter and maximize according to it. This procedure has been taken by Ravn et al. (2004). However the problem does not have any analytical solution in such a situation; it is solved by computer simulation. For the sake of simplicity and clarity we assume that individuals know the evolution of technology even though we allow them no control over the habit formation process. The notion is that an individual's decision to buy is based on past experience but in this decision they do not take into consideration that the present choice will affect their future preferences. In this sense the individual is myopic. However, individuals adjust their savings because they know the flow of innovations will provide superior products in the future.

The separability assumption is also used by (Grossman and Helpman, 1991, Chapter 3), and Foellmi and Zweimuller (2004). They work with a static demand, the situation that we shall arrive at since we impose a sufficiently large number of within subperiods to allow us to include the same assumptions.

For simplicity and without lost of generality, it is assumed that the population is constant and that each individual represents an adult working member of the current generation. Thus there is a fixed and large number (normalized to be one) of identical adults who take into account the welfare and resources of their actual and prospective descendants. Indeed, following Barro and Sala-i Martin (1995), this intergenerational interaction is modelled by imagining that the current generation maximizes utility and incorporates a budget constraint over an infinite future. That is, although individuals have finite lives, the model considers immortal extended families ("dynasties"). We do not allow individuals to engage in Ponzi games. In the first step individuals reach the within periods solution; in the second step they maximize intertemporal utility.

Step 1 The problem for the representative agent within the period is:

$$\underset{x_{i\tau}}{Max} \qquad (\sum_{\text{i=1}}^{\text{t}} (x_{i\tau} - c_i x_{i\tau-1})^{\alpha})^{1/\alpha} \tag{4}$$

$$s.t \qquad \sum_{\text{i=1}}^{\text{t}} x_{i\tau} p_{i\tau} = E_t \tag{5}$$

The individual can only choose her actual consumption, how much of each good she wants according to the price, such that $x_{i\tau}$ is the quantity of the good i in the subperiod τ. The consumer is too small to influence the price; she is a price taker. Within the periods the budget is constant. The end of the sum is t, because the number of varieties present in each within period is t. Only between periods will budgets change; this will be explained later. The consumer decides on the actual quantity she wants to have for this period. She is myopic in the sense that she does not maximize controlling for the effect that the present consumption might have in the future[5]. This assumption is made, as previously explained, to be able to find an analytical solution avoiding simulations. For the changes in time between periods the letter t is used and for changes within periods we use τ. We sum across goods because in each innovation lifespan interval the number of goods is equal to t.

For the solution in the long-term of the within period, τ tends to t:[6]

$$x^*_{it} = \frac{\frac{p_{it}^{\xi}}{(1-c_i)}}{\sum_{\text{j=1}}^{\text{t}} \frac{p_{jt}^{\xi+1}}{(1-c_j)}} E_t \tag{6}$$

with $\xi = \frac{1}{\alpha-1}$ and $0 < \alpha < 1$, accounting for the elasticity of substitution between the individual's past experienced utility consuming good i, $c_i x_{i\tau-1}$. and her present desired consumption level.

[5] In mathematical terms she is maximizing her utility according to $x_{i\tau}$, she has no control in τ over $x_{i\tau-1}$

[6] This is a similar solution to that obtained in the previous article presented by equation number **??**

The solution within periods could be written as follows, knowing how parameter γ determines the relation in the flow of innovations and that equation 1 defines how one innovation relates to the previous one:

$$x^*_{it} = \frac{\gamma^{i-1} p_{it}^{\xi}}{\sum\limits_{j=1}^{t} \gamma^{j-1} p_{jt}^{\xi+1}} E_t \tag{7}$$

The income elasticity within the period is equal to[7]:

$$\varepsilon_{Et} = \frac{dx_{it}}{dE_t} \frac{E_t}{x_{it}} = 1$$

Step 2 The individual faces the following intertemporal maximization or maximization between time steps:

$$\max \sum_{t=1}^{\infty} \beta^t U_t \tag{8}$$

The parameter β controls the discounting of utility over time, taking values between zero and one. The closer it gets to zero, the less the individual values future consumption. By assumption new innovations only arrive when the consumer is already in equilibrium within the period and, taking into consideration the evolution of technology presented by equation 1, the maximization can be rewritten in the following way:

$$\max_{\substack{(x_{1t}.....x_{tt}) \\ s_t}} \quad \sum_{t=1}^{\infty} \beta^t ((1-c_1)(\sum_{i=1}^{t} \gamma^{(i-1)\alpha} x_{it}^{\alpha}))^{1/\alpha} \tag{9}$$

$$s.t. \quad \sum_{i=1}^{t} x_{it} p_{it} + s_t = d_t + \Pi_t + w_t \tag{10}$$

$$d_{t+1} = (1 + r_t) s_t \tag{11}$$

$$\lim_{t \to \infty} \lambda_t d_t = 0 \tag{12}$$

where $\sum\limits_{i=1}^{t} x_{it} p_{it}$ is the individual's total expenditure in time t and s_t the amount she wants to save in the same period. This decision is taken at the beginning of each period of time, with expenditures and savings being readjusted each new period. Individual savings of assets d_t are in the form

[7]

$$\varepsilon_{Et} = \frac{dx_{it}}{dE_t} \frac{E_t}{x_{it}} = \frac{\gamma^{i-1} p_{it}^{\xi}}{\sum\limits_{j=1}^{t} \gamma^{j-1} p_{jt}^{\xi+1}} \frac{E_t}{\frac{\gamma^{i-1} p_{it}^{\xi}}{\sum\limits_{j=1}^{t} \gamma^{j-1} p_{jt}^{\xi+1}} E_t} = 1$$

of ownership of firms or loans. Negative loans represent debts. Households can lend or borrow from other households but in equilibrium the net loans of a representative consumer will be zero. Because both forms of assets, shares and loans, are assumed to be perfect substitutes as stores of value they must pay the same real rate of return r_t. As income at the beginning of the period, an individual has the value of their assets d_t, the salary w_t and the redistribution of profits in form of dividends Π_t. The level of savings in this period is determined, depending on r_t, by the amount of assets they wish to hold at the beginning of the next period d_{t+1}.

Using the definition of the indirect utility function we arrive to the following general Euler rule (see appendix **??**):

$$\frac{\left(\sum_{i=1}^{t+1}\left(\frac{\gamma^{i-1}p_{it}^{\xi}}{\sum_{i=1}^{t+1}\gamma^{i-1}p_{it}^{\xi+1}}\right)^{\alpha}\right)^{1/\alpha}}{\left(\sum_{i=1}^{t}\left(\frac{\gamma^{i-1}p_{it}^{\xi}}{\sum_{i=1}^{t}\gamma^{i-1}p_{it}^{\xi+1}}\right)^{\alpha}\right)^{1/\alpha}} = \frac{1}{(1+r_t)\beta} \tag{13}$$

This equation explains how the consumer decides to allocate resources over time. Consumers choose quantities but, because we have used the indirect utility function, the solution's quantities are written in terms of prices. Prices are determined by firms; this is discussed in the next subsection. The first fraction relates the consumption of next year to that of this year. The increase or decrease depends on the interest rate and on the β parameter. Changes on these parameters will affect to the selected quantity of each good in each period. If the interest rate rises consumers will prefer to save more for the future. The discount rate for future consumption is represented in our model by the parameter β; as it increases future consumption will have a greater weight in the present decision and present consumption will decrease.

We choose the numeraire in such a way that expenditure on the first good is always equal to one in each period,

$$p_{1t}x_{1t} = 1 \ \forall t.$$

Making use of the solution 7 and the solution of prices from the next section, the Euler rule can be expressed as[8]:

$$\left(\frac{1+\gamma^{\alpha}+...+\gamma^{\alpha(t)}}{1+\gamma^{\alpha}+...+\gamma^{\alpha(t-1)}}\right)^{1/\alpha}\frac{E_t}{E_{t+1}}\frac{p_t}{p_{t+1}} = \frac{1}{(1+r_t)\beta}$$

[8]For technical notes dealing with the mathematical steps see appendix **??**

4.3 The production side of the economy

Two different sectors interact in the economy, one sector producing final goods in a monopolistic competence market and another sector producing blueprints.

4.3.1 Firms producing final goods

Each final goods firm produces a different good using as input both labour and a blueprint for a product design. This latter has been bought from the R&D sector, which has patented the rights to this innovation. Product designs are assumed to be proprietary information either because their details can be kept secret or because patents effectively deter unauthorised use. The production firm starts exploiting monopoly rights and earning monopoly profits in the period in which they buy the patent.

Each known variety is produced by a different single atomistic firm. This assumption can be justified in one of two ways. First, the government may grant long-term patents to the original inventors of innovative products. Alternatively, we may suppose that imitation is costly and that firms engage in ex-post price competition. In this case, no entrepreneur would ever invest resources to copy a brand that is already available on the market. A copier would earn zero profits in Bertrand competition with the original innovator and so would be unable to recoup a positive cost of imitation.

Each individual firm behaves as a monopolist in imperfectly competitive markets. Latecomers, because they buy a better brand, will derive higher profits. By assumption firms have no control over habit formations. Each firm maximizes its profit in each period; the optimal solution is for the firm to charge monopoly prices. Firms are sufficiently small so as not to influence other firms' prices. (The subsection on behaviour of firms parallels that presente by (Grossman and Helpman, 1991, Chapter 3, p.49-52)). We assume that all products are manufactured subject to a common constant returns to scale technology.

The production function of the firm is:

$$x_{it} = aL_{it} \tag{14}$$

All firms have the same production technology, which remains constant in all periods and is equal to a. This parameter accounts for the marginal cost of production of each unit of good. It is assumed that the parameter accounts for paying back the entrepreneur in some way. Each unit of labour L_{it} is paid at the wage rate w_t.

Assuming that firms maximize profits in the framework of monopolistic competition, we can define a firm's profit as:

$$\pi_{it} = p_{it}x_{it} - \frac{w_t}{a}x_{it}$$

With the solution of the static utility for the prices (see Appendix **??**) and the solution obtained by equation **??** we arrive at: [9]

$$\pi_{it} = \frac{\gamma^{(i-1)(\alpha-1)} \quad x_{it}^{\alpha-1}}{\sum_{j=1}^{t} \gamma^{(j-1)(\alpha-1)} \quad x_{jt}^{\alpha}} x_{it} E_t - \frac{w_t}{a} x_{it}$$

or

$$\pi_{it} = \frac{\gamma^{(i-1)(\alpha-1)} \quad x_{it}^{\alpha}}{\sum_{j=1}^{t} \gamma^{(j-1)(\alpha-1)} \quad x_{jt}^{\alpha}} E_t - \frac{w_t}{a} x_{it}$$

We will assume that the number of firms is big enough for a decision taken by one firm to have no influence on any other firm[10]. Calculating the derivative with respect to the quantity we find:

$$\frac{\partial}{\partial x_{it}} \pi_{it} = \alpha \frac{\gamma^{(i-1)(\alpha-1)} \quad x_{it}^{\alpha-1}}{\sum_{j=1}^{t} \gamma^{(j-1)(\alpha-1)} \quad x_{jt}^{\alpha}} E_t - \frac{w_t}{a} = 0$$

Taking into account the definition of the prices given by equation **??** and solving for p_{it} we have:

$$p_{it} = \frac{w_t}{a\alpha} \tag{16}$$

Using the Amaroso-Robinson condition that prices are markups of costs:

$$\pi_{it} = \frac{w_t}{a} (\frac{1-\alpha}{\alpha}) x_{it} \tag{17}$$

An important conclusion is that prices are independent of the quantities sold and also independent of the γ parameter. Prices will be equal for all goods and they increase at the same rate as wages. However, profits are dependent on the parameter γ through x_{it} which changes with time, meaning with each new innovation arrival.

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[9] In subsection 4.2 we have done exactly the same but the solution was presented as the resultant quantity in function of the prices. For the convenience of the algebra we express prices in function of the quantities consumed. To arrive at this solution see Appendix **??**

$$p_{it} = \frac{\gamma^{(i-1)(\alpha-1)} \quad x_{it}^{\alpha-1}}{\sum_{j=1}^{t} \gamma^{(j-1)(\alpha-1)} \quad x_{jt}^{\alpha}} E_t \tag{15}$$

[10] We assume that the economy starts from a situation in which there is one theoretical single good x_{11}. This good represents a set of goods all with equal habit formation parameter c_1. In this sense we start from a stable solution similar to the final solution presented by (Grossman and Helpman, 1991, p. 43-54). In their model they increase variety but all goods are symmetric, a stable solution in which no economic growth is discussed. Our initial good, x_{11}, may be thought of as their resultant set of goods.

$$\pi_{it} = w_t(\frac{1-\alpha}{\alpha})L_{it} \tag{18}$$

Now we can study the distribution of labour among final goods firms.

Using the solution presented by Eq 7 we can study the relation between quantities x^*_{1t} and x^*_{2t}, where the stars stand for the long-term solution of the within period:

$$x^*_{1t} = \frac{p^{\xi}_{1t}}{\sum_i \gamma^{(i-1)} p^{\xi+1}_{it}} E_t$$

$$x^*_{2t} = \frac{\gamma p^{\xi}_{2t}}{\sum_i \gamma^{(i-1)} p^{\xi+1}_{it}} E$$

Since prices are equal for all goods (solution 16) in each period:

$$x^*_{1t} = \frac{p^{\xi}_{t}}{\sum_i \gamma^{(i-1)} p^{\xi+1}_{t}} E_t$$

$$x^*_{2t} = \frac{\gamma p^{\xi}_{t}}{\sum_i \gamma^{(i-1)} p^{\xi+1}_{t}} E_t$$

$$x^*_{2t} = \gamma x^*_{1t}$$

Having a single linear production function (equation 14), similar for all final good sectors, and knowing the relation between the quantities produced in each interval given by solution 7, we can arrive at the distribution of labour across final goods:

$$\begin{aligned}
x^*_{1t} &= aL_{1t} \\
x^*_{2t} &= aL_{2t} = \gamma a L_{1t} \\
x^*_{3t} &= aL_{3t} = \gamma^2 a L_{1t} \\
\ldots & \\
x^*_{it} &= aL_{it} = \gamma^{(i-1)} a L_{1t} \\
L_{FG\,t} &= \sum_i L_{it}
\end{aligned}$$

And the distribution of labour for each sector will be:

$$L_{it} = \frac{\gamma^{i-1}}{\sum_{i=1}^{t} \gamma^{i-1}} L_{Final\ Goods} \tag{19}$$

where $L_{Final\ Goods}$ or L_{FG} will be the total amount of labour devoted by the economy to the production of final goods. We will assume that the system is in equilibrium first, therefore we know how many workers are devoted to final production and how many to R&D is a constant proportion. When referring to individual goods, in each period there will be a different amount of labour and the time subscript is needed. Taking as an example the amount of labour employed by the good 2 in the periods $t = 3$ and $t = 4$:

$$L_{23} = \frac{\gamma}{1+\gamma+\gamma^2} L_{Final\ Goods}$$
$$L_{24} = \frac{\gamma}{1+\gamma+\gamma^2+\gamma^3} L_{Final\ Goods}$$

The individual firm's profit will be equal to:

$$\pi_{it} = w_t(\frac{1-\alpha}{\alpha})\frac{\gamma^{i-1}}{\sum\limits_{i=1}^{t} \gamma^{i-1}} L_{Final\ Goods} \tag{20}$$

and the total profits in the economy:

$$\Pi_t = w_t(\frac{1-\alpha}{\alpha}) L_{Final\ Goods} \tag{21}$$

It is interesting to study the relation between quantities produced and profits as time evolves.

Sector	Time 1	2	3	[...]	t	t+1	[...]
1	$x_{1\,1}$	$x_{1\,2}$	$x_{1\,3}$	...	$x_{1\,t}$	$x_{1\,t+1}$	...
2		$x_{2\,2}$	$x_{2\,3}$	...	$x_{2\,t}$	$x_{2\,t+1}$	...
3			$x_{3\,3}$	...	$x_{3\,t}$	$x_{3\,t+1}$	...
[...]	...	...	...	...	...	...	...
i				...	$x_{i\,t}$	$x_{i\,t+1}$	...
i+1				...		$x_{i+1\,t+1}$	...
[...]	...	...	...	...	...	...	...

Table 1: Production of quantities for each good in each period of time

Table 1 attempts to describe what is happening in this economy. In the first period there is one single good $x_{1\,1}$. All labour devoted to final goods will be busy producing this single good. In the next period a new good, $x_{2\,2}$, will appear and a proportion of the labour which was earlier producing good 1 will shift to the production of good 2. If the system is in equilibria in the distribution of labor, the total quantity of labour devoted to final goods and the production coefficients are fixed, the total quantity of good 1 produced in time period 2 will diminish to $x_{1\,2}$. The table presents a diagonal structure, a consequence of the idea that in each time period an innovation is introduced and produced by the economy. Using some definitions given by previous formulas (see Appendix **??**) the evolution of profits over time as new goods appear is represented in the following table (2). In the next section we will explain that the value of the patent depends on the flow of profits, therefore a similar table will be introduced to explain the relation between the value of a patent and that of the subsequent one.

Sector	**Time** **1**	**2**	**3**	**[...]**	**t**	**t+1**	**[...]**
1	$w_1(\frac{1-\alpha}{\alpha})L_{FG}$	$\frac{1}{1+\gamma}\ w_2(\frac{1-\alpha}{\alpha})L_{FG}$	$\frac{1}{1+\gamma+\gamma^2}\ w_3(\frac{1-\alpha}{\alpha})L_{FG}$	...	$\frac{1}{\sum_{i=1}^{i=t}\gamma^{(i-1)}}\ w_t(\frac{1-\alpha}{\alpha})L_{FG}$	$\frac{1}{\sum_{i=1}^{i=t+1}\gamma^{(i-1)}}\ w_{t+1}(\frac{1-\alpha}{\alpha})L_{FG}$	$\dots$
2		$\frac{\gamma}{1+\gamma}\ w_2(\frac{1-\alpha}{\alpha})L_{FG}$	$\frac{\gamma}{1+\gamma+\gamma^2}\ w_3(\frac{1-\alpha}{\alpha})L_{FG}$	$\dots$	$\frac{\gamma}{\sum_{i=1}^{i=t}\gamma^{(i-1)}}\ w_t(\frac{1-\alpha}{\alpha})L_{FG}$	$\frac{\gamma}{\sum_{i=1}^{i=t+1}\gamma^{(i-1)}}\ w_{t+1}(\frac{1-\alpha}{\alpha})L_{FG}$	$\dots$
3			$\frac{\gamma^2}{1+\gamma+\gamma^2}\ w_3(\frac{1-\alpha}{\alpha})L_{FG}$	$\dots$	$\frac{\gamma^2}{\sum_{i=1}^{i=t}\gamma^{(i-1)}}\ w_t(\frac{1-\alpha}{\alpha})L_{FG}$	$\frac{\gamma^2}{\sum_{i=1}^{i=t+1}\gamma^{(i-1)}}\ w_{t+1}(\frac{1-\alpha}{\alpha})L_{FG}$	$\dots$
[...]	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
i				$\dots$	$\frac{\gamma^{i-1}}{\sum_{i=1}^{i=t}\gamma^{(i-1)}}\ w_t(\frac{1-\alpha}{\alpha})L_{FG}$	$\frac{\gamma^{i-1}}{\sum_{i=1}^{i=t+1}\gamma^{(i-1)}}\ w_{t+1}(\frac{1-\alpha}{\alpha})L_{FG}$	$\dots$
i+1				$\dots$		$\frac{\gamma^{i}}{\sum_{i=1}^{i=t+1}\gamma^{(i-1)}}\ w_{t+1}(\frac{1-\alpha}{\alpha})L_{FG}$	$\dots$
[...]	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$

Table 2: Evolution of profits

4.3.2 The R&D sector

We treat R&D as an ordinary economic activity that requires input resources. The sector produces blueprints using only labour as input. The production function has constant returns to scale in the number of blueprints produced although each blueprint will have a higher value than the previous one. Each blueprint is sold to a monopolist that will exploit its monopoly income from the innovation in the final goods market. The value of each innovation is the sum of the discounted future profits. In each period an innovation materialises and there is a new asset in the economy.

The R&D sector has incomes that depend on the value of newly created innovations. The value of an innovation is the present value of the future profits generated by this specific innovation. Every time a new blueprint is bought a whole new sector (i) starts producing:

$$d_{i+1} = \frac{\pi_{i+1\ t+1}}{(1+r_{t+1})} + \frac{\pi_{i+1\ t+2}}{(1+r_{t+1})(1+r_{t+2})} + \frac{\pi_{i+1\ t+3}}{(1+r_{t+1})(1+r_{t+2})(1+r_{t+3})} + ... + 0$$
$$d_{i+1} = \sum_{j=1}^{\infty} \frac{\pi_{i+1\ t+j}}{\prod_{\iota=1}^{\iota=j}(1+r_{t+\iota})}$$

The relationship between the value of a patent and that of the subsequent patent can be easily seen:

$$d_{i+1} = \gamma d_i$$

As an example we compare the value of two patents d_1 and d_2 in $t = 2$. Table 3 is a reformulation of the previous table 2. Comparing the two highlighted rows we see that in $t = 2$ the present value of the flow of profits for the second good is γ times superior to the discounted value of the profits for the first good:

$$d_2 = \gamma d_1$$

Sector	Time 1	2	3	[...]	t	t+1	[...]
1		d_1	$\frac{1}{1+\gamma+\gamma^2}\ w_3(\frac{1-\alpha}{\alpha})L_{FG3}$	...	$\frac{1}{\sum_{i=1}^{i=t}\gamma^{(i-1)}}\ w_t(\frac{1-\alpha}{\alpha})L_{FG}$	$\frac{1}{\sum_{i=1}^{i=t+1}\gamma^{(i-1)}}\ w_{t+1}(\frac{1-\alpha}{\alpha})L_{FG}$	...
2		d_2	$\frac{\gamma}{1+\gamma+\gamma^2}\ w_3(\frac{1-\alpha}{\alpha})L_{FG3}$	...	$\frac{\gamma}{\sum_{i=1}^{i=t}\gamma^{(i-1)}}\ w_t(\frac{1-\alpha}{\alpha})L_{FG}$	$\frac{\gamma}{\sum_{i=1}^{i=t+1}\gamma^{(i-1)}}\ w_{t+1}(\frac{1-\alpha}{\alpha})L_{FG}$	...
3			$\frac{\gamma^2}{1+\gamma+\gamma^2}\ w_3(\frac{1-\alpha}{\alpha})L_{FG3}$	...	$\frac{\gamma^2}{\sum_{i=1}^{i=t}\gamma^{(i-1)}}\ w_t(\frac{1-\alpha}{\alpha})L_{FG}$	$\frac{\gamma^2}{\sum_{i=1}^{i=t+1}\gamma^{(i-1)}}\ w_{t+1}(\frac{1-\alpha}{\alpha})L_{FG}$	...
[...]	...	...	...	...	...	...	...
i				...	$\frac{\gamma^{i-1}}{\sum_{i=1}^{i=t}\gamma^{(i-1)}}\ w_t(\frac{1-\alpha}{\alpha})L_{FG}$	$\frac{\gamma^{i-1}}{\sum_{i=1}^{i=t+1}\gamma^{(i-1)}}\ w_{t+1}(\frac{1-\alpha}{\alpha})L_{FG}$	...
i+1				...		$\frac{\gamma^{i}}{\sum_{i=1}^{i=t+1}\gamma^{(i-1)}}\ w_{t+1}(\frac{1-\alpha}{\alpha})L_{FG}$	...
[...]	...	...	...	...	...	...	...

Table 3: Comparison for the value of patents

We now concentrate on the production in the R&D sector which in every period of time produces a single blueprint using as unique resource a fixed and constant amount of labour. Each produced blueprint has its own different value which depends on the future profits of the innovation. The total profit of the sector will depend on revenues d_{i+1} and total costs of producing the innovation will depend on the total labour used and on the salary level $w_t L_{R\&D}$. Even though we have so far assumed that the system is in equilibria, a higher amount of labor in the sector is related to a superior utility jump. In subsection 4.1 when the arrival of innovation in relation to time was explained the equation , gives the necessary relation between the c_i, so that the utility jump given by the last good is γ times bigger in relation to the previous good,

$$c_{i+1} = \frac{\gamma + c_i - 1}{\gamma} \tag{22}$$

Let us compare the situations of two hypothetical closed economies with the same population each of them making a different technological effort. The first economy (1), having a total amount of labor in the R&D equals to $L^{(1)}_{R\&D}$, and the second economy (2) doing a superior effort $L^{(2)}_{R\&D}$.

$$L^{(1)}_{R\&D}: \quad c^{(1)}_1 \xrightarrow{\gamma^{(1)}} c^{(1)}_2 \xrightarrow{\gamma^{(1)}} ....c^{(1)}_t \xrightarrow{\gamma^{(1)}} c^{(1)}_{t+1}$$

$$L^{(2)}_{R\&D}: \quad c^{(2)}_1 \xrightarrow{\gamma^{(2)}} c^{(2)}_2 \xrightarrow{\gamma^{(2)}} ....c^{(2)}_t \xrightarrow{\gamma^{(2)}} c^{(2)}_{t+1}$$

If one economy has more people working during the same period of time, the flow of innovations generated by this economy will have associated higher utility jumps. If γ is measuring this jump, it is expected that if $L^{(1)}_{R\&D} < L^{(2)}_{R\&D}$ then $\gamma^{(1)} < \gamma^{(2)}$. In other words superior innovative effort will be related to a superior flow of innovations which will be associated to superior increases in utility.

The production function of the R&D sector is relating inputs to outputs. Our only input is labour and by the assumptions in the utility side we know that the output per period will be always equal to one. Only one innovation at the time is possible and the output of the R&D will be always an idea. At the macroeconomic level however superior economic efforts are related to superior outputs, this relation is not a normal production function, but a translation of the research effort made by the economy, this idea is capture by the next equation

$$\gamma = \phi L_{R\&D} \tag{23}$$

Where the parameter ϕ transforms the innovative inputs of the economy $L_{R\&D}$ during a period to the total impact that such innovation will have in the economy γ. However equation 23 is not a normal production function, but a relation between the effort done and the impact that this effort has in the economy.

After we have discussed this issue we present the profits of the R&D sector. In each period of time the sector will produce a new innovation, that will have a value of d_{i+1}. This new innovation as it has been explained in will be γ times bigger than the previous one, therefore the total ravenue of the R&D sector can be written as:

$$d_{i+1} = \gamma d_i = \phi L_{R\&D} d_i$$

And the total profit, will be total ravenues minus total costs $w_t L_{R\&D}$,

$$\phi L_{R\&D} d_i - w_t L_{R\&D} \tag{24}$$

The sector will maximize the profits making marginal revenues equal marginal costs,

$$\phi d_i = w_t \tag{25}$$

Rearranging terms:

$$d_i = w_t / \phi \tag{26}$$

4.4 The equity market

Equity holders expect to enjoy capital gains (or suffer capital losses) on their ownership shares. In a perfect-foresight equilibrium these expected gains or losses must match the change that actually occurs in the value of the firm. We let d_t denote the value of a claim to the infinite stream of profits that accrues to a newly invented innovation. If such an amount is invested in the R&D sector the arbitrage in capital markets ensures equality between this yield and that from a similar amount on a riskless loan. The return of an investment on a riskless loan of size d_t will be $d_t r_t$. The expected gains in the sector producing blueprints will be a new patent d_{i+1} and the profit associated with this patent π_{i+1}. The arbitrage condition of the equity market will be:

$$d_i r_t = d_{i+1} - d_i + \pi_{i+1}$$

which means that, if we save one unit d_i, the increase in the value of the savings $d_i r_t$ has to be equal to the value generated by an investment of the same unit d_i made in the R&D sector $(d_{i+1} + \pi_{i+1})$. In equilibrium the value generated by investments in R&D must be equal to the value generated by a riskless loan.

5 The general equilibrium

We define a balanced growth path as the moment at which wages and prices (and therefore expenditures) grow at the same stable rate. This constant rate will be the growth rate of the economy in the balanced growth path. Before continuing with the solutions we recall the results already presented in the previous section:

1. We start with the Euler rule presented by equation 13. From the solution presented in equation 16 we know that prices are equal across goods in each time period. We will choose the numeraire so that expenditure on the initial quantity will be equal to one in every period:

   $$p_{1t}x_{1t} = 1 \quad \forall t.$$

   Taking into consideration these two previous equations, equation 13 can be written as (see appendix **??**):

   $$\left(\frac{1+\gamma^{\alpha}+...+\gamma^{\alpha(t)}}{1+\gamma^{\alpha}+...+\gamma^{\alpha(t-1)}}\right)^{1/\alpha}\frac{E_t}{E_{t+1}}\frac{p_t}{p_{t+1}} = \frac{1}{(1+r_t)\beta}$$

   Solving for r_t,

   $$r_t = \frac{1}{\beta\left(\frac{1+\gamma^{\alpha}+...+\gamma^{\alpha(t)}}{1+\gamma^{\alpha}+...+\gamma^{\alpha(t-1)}}\right)^{1/\alpha}}\frac{E_{t+1}}{E_t}\frac{p_{t+1}}{p_t} - 1 \tag{27}$$

2. From the maximization of profits equation 20 we have:

   $$\pi_{it} = w_t\left(\frac{1-\alpha}{\alpha}\right)\frac{\gamma^{i-1}}{\sum_{i=1}^{t}\gamma^{i-1}}L_{Final\ Goods}$$

   The profit of the last good invented in the subsequent period will be $i = t+1$, in $t+1$

   $$\pi_{i+1,t+1} = w_{t+1}\left(\frac{1-\alpha}{\alpha}\right)\frac{\gamma^{t}}{\sum_{i=1}^{t+1}\gamma^{i-1}}L_{Final\ Goods} \tag{28}$$

3. From the labour market condition we find that the total labour of the economy has to equal the number of people employed in each sector:

   $$L = L_{R\&D} + L_{FG}$$

   Normalizing total labour to one we have that:

   $$1 = L_{R\&D} + L_{FG} \tag{29}$$

4. Taking the labour constraint into consideration, maximizing the R&D sector as shown in equation 26 gives:

$$d_i = w_t/\phi \tag{30}$$

Now we start with the arbitrage condition of the equity markets:

$$d_i r_t = d_{i+1} - d_i + \pi_{i+1}$$

Solving for r_t we have

$$r_t = \frac{d_{i+1} - d_i}{d_i} + \frac{\pi_{i+1}}{d_i}$$

Knowing that each innovation is γ times superior to the previous one:

$$r_t = \frac{\gamma d_i - d_i}{d_i} + \frac{\pi_{i+1}}{d_i}$$

$$r_t = (\gamma - 1) + \frac{\pi_{i+1}}{d_i} \tag{31}$$

Substituting in equation 31 the solution for r_t that comes from the value obtained by the Euler rule (equation 27), the solution from the R&D sector 30, and the maximization of profits from final goods (equation **??**) produces:

$$\frac{1}{\beta\left(\frac{1+\gamma^{\alpha}+...+\gamma^{\alpha(t)}}{1+\gamma^{\alpha}+...+\gamma^{\alpha(t-1)}}\right)^{1/\alpha}}\frac{E_{t+1}}{E_t}\frac{p_{t+1}}{p_t} - 1 = (\gamma - 1) + \frac{w_t \frac{E_{t+1}}{E_t}\left(\frac{1-\alpha}{\alpha}\right)\frac{\gamma^t}{\sum_{i=1}^{t+1}\gamma^{i-1}} L_{FG}}{w_t/\phi} \tag{32}$$

Having defined the balanced growth path (as the moment at which wages and prices grow at the same stable rate), when t goes to infinity we can introduce the value of the limits given by solutions **??** and **??** (see appendix **??**). The only thing constantly growing in each period is the total value of the patents which, using the results of section 4.3.2, is:

$$\frac{d_{i+1} - d_i}{d_i} = \frac{\gamma d_i - d_i}{d_i} = (\gamma - 1)$$

Since the number of people devoted to final goods is constant, total expenditures can be expressed as:

$$E_t = p_t X^{**} = p_t a L_{FG}^{**}$$

Calculating the growth rate of expenditures:

$$\frac{E_{t+1} - E_t}{E_t} = \frac{p_{t+1} a L_{FG}^{**} - p_t a L_{FG}^{**}}{p_t a L_{FG}^{**}} = \frac{p_{t+1} - p_t}{p_t} = (\gamma - 1)$$

When we are in the long-term balanced growth path we have:

$$\frac{w_{t+1} - w_t}{w_t} = \frac{p_{t+1} - p_t}{p_t} = \frac{E_{t+1} - E_t}{E_t} = \frac{d_{t+1} - d_t}{d_t} = \gamma - 1$$

and $\frac{E_{t+1}}{E_t} = \frac{p_{t+1}}{p_t} = \gamma$

Solving the previous equation 32 in the long run(using the limits of appendix **??**), substituting $\gamma = \phi L_{R\&D} = \phi(1 - L_{FG})$ and rearranging terms provides the following equation:

$$L_{FG}^2 - \frac{\phi - 1}{\phi} L_{FG} + \frac{(1-\beta)}{\beta} \frac{\alpha}{(1-\alpha)\phi^2} = 0 \tag{33}$$

Solving for L_{FG}

$$L_{FG}^{**} = \frac{\frac{\phi-1}{\phi} + \sqrt{\left(\frac{\phi-1}{\phi}\right)^2 - 4\frac{(1-\beta)}{\beta}\frac{\alpha}{(1-\alpha)\phi^2}}}{2} \tag{34}$$

This solution gives us the proportion of labor that will assure a constant rate of growth, which will be equal to,

$$g^{**} = (\gamma - 1) = \phi L_{R\&D}^{**} - 1 = \phi(1 - L_{FG}^{**}) - 1 \tag{35}$$

5.1 Transition to the equilibrium

So far, we have assumed that there is a constant proportion of labour in each sector, in this subsection we study transition to the equilibria. By assuming a constant proportion of labour in the R&D sector in all periods, we were getting a constant γ. If the economy starts from a distribution of labour different than the one given by equation 34, it will mean that in the first periods while the labour is adjusting we will get a different γ. Let us analyse a situation in which the labour in the R&D sector is different than the one in equilibria, assuming that there is free mobility of labour between the two sectors equation tell us that the in a moment of time in which the marginal productivity of labour in the two sectors will be equal, If the economy starts from a situation in which $L^t_{R\&D} \neq L^{**}_{R\&D}$, and we will assume that it takes n periods to reach the equilibrium, in such a situation then we will have an array of gammas like,

$$
\begin{aligned}
&\gamma_{(2)} = \phi L_{R\&D,t=2} \\
&\gamma_{(3)} = \phi L_{R\&D,t=3} \\
&.... \\
&\gamma_{(n)} = \phi L_{R\&D,t=n} \\
&\gamma_{(**)} = \phi L_{R\&D,t=n+1} \\
&.... \\
&\gamma_{(**)} = \phi L_{R\&D,t=\infty}
\end{aligned}
$$

Where γ_{**} is the parameter that corresponds to $L^{**}_{R\&D}$. We discuss now how our equilibrium will be affected by such a change, we focus on equation 32, and in the changes that different γs will have,

$$\frac{1}{\beta\left(\frac{1+\gamma^{\alpha}+...+\gamma^{\alpha(t)}}{1+\gamma^{\alpha}+...+\gamma^{\alpha(t-1)}}\right)^{1/\alpha}}\frac{E_{t+1}}{E_t}\frac{p_{t+1}}{p_t} - 1 = (\gamma - 1) + \frac{w_t \frac{E_{t+1}}{E_t}(\frac{1-\alpha}{\alpha})\frac{\gamma^t}{\sum_{i=1}^{t+1}\gamma^{i-1}}L_{FG}}{w_t/\phi}$$

In this equation, the variation introduce will affect the fractions $\left(\frac{1+\gamma^{\alpha}+...+\gamma^{\alpha(t)}}{1+\gamma^{\alpha}+...+\gamma^{\alpha(t-1)}}\right)^{1/\alpha}$ and $\frac{\gamma^t}{\sum_{i=1}^{t+1}\gamma^{i-1}}$ which will be then be transformed into,

$$\left(\frac{1+\gamma^{\alpha}_{(2)} + (\gamma_{(2)}\gamma_{(3)})^{\alpha}... + (\gamma_{(2)}\gamma_{(3)}...\gamma_{(n)}\gamma^{(t-n)\alpha}_{(**)})}{1+\gamma^{\alpha}_{(2)} + (\gamma_{(2)}\gamma_{(3)})^{\alpha}... + (\gamma_{(2)}\gamma_{(3)}...\gamma_{(n)}\gamma^{(t-n-1)\alpha}_{(**)})}\right)^{1/\alpha} \tag{36}$$

$$\left(\frac{(\gamma_{(2)}\gamma_{(3)}...\gamma_{(n)}\gamma^{(t-n)}_{(**)})}{1+\gamma_{(2)} + (\gamma_{(2)}\gamma_{(3)})... + (\gamma_{(2)}\gamma_{(3)}...\gamma_{(n)}\gamma^{(t-n-1)}_{(**)})}\right) \tag{37}$$

In appendix **??** we calculate the value of this two limits which are equal to $\gamma_{(**)}$ and $\gamma_{(**)} - 1$. Therefore such variation will not affect the general equilibria, since the value of the limits is not affected by this change, neither will it be any of the other assumptions or relations presented in the previous section. Having different γs will only make it harder to follow up what is happening in the model.

5.2 Analysis of the parameter of the model

In this subsection we explain how will the change in the value of the parameter affect our solution, we will focus on the solution of $L^{**}_{R\&D}$ and the general growth of the economy g^{**}.

$$L^{**}_{R\&D} = 1 - \frac{\frac{\phi-1}{\phi} + \sqrt{\left(\frac{\phi-1}{\phi}\right)^2 - 4\frac{(1-\beta)}{\beta}\frac{\alpha}{(1-\alpha)\phi^2}}}{2} \tag{38}$$

$$g^{**} = (\gamma - 1) = \phi L^{**}_{R\&D} - 1 \tag{39}$$

The next two figures 3 represents the value of the distribution of labour in R&D and growth in equilibrium, for different parameters. To curves are represented in a 3 dimensional plot for $\alpha = 0.5$ and $\alpha = 0.75$. In table 4 a calibration of the value of the parameters for $\alpha = 0.5$ is presented. In graphs in can be observed that higher values of α, with everything else constant translates into higher values of growth. The parameter α is related to the elasticity of substitution of the CES function, the higher this parameter the easier that consumers will switch to new innovations. Our β is related to the consume's preferences of present consumption versus future consumption. The closer this parameter is to one, the lower the growth rate of the economy. Because the lower this parameter is the less consumers will want to go for present consumption, and will postpone consumption for future periods. The last parameter ϕ affects negatively to growth, this parameter regulates the marginal productivity of labour in the R&D, if the parameter is higher it will be need it less labour in this sector that will go to the production of final goods. The change in this parameter affects in two ways to the growth rate, since the growth rate of the economy is the one presented by equation 39. If $\phi \uparrow$ goes up , the quantity will be affected by two movements $\phi(\uparrow) L^{**}_{R\&D}(\downarrow) - 1$, resulting in a total decreasing effect that dominates. This is the case in the range for which the parameters have an economic value. All this effect can be observed in the graphs and tables.

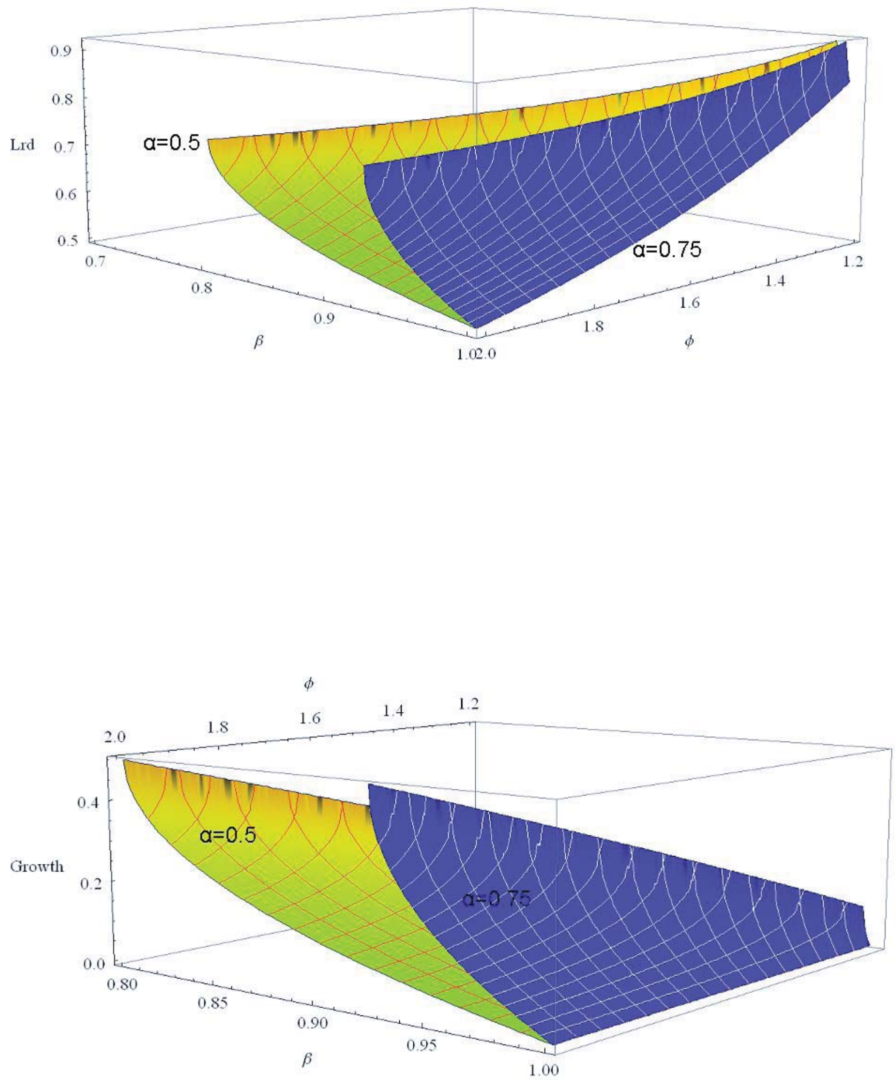

Figure 3: Value of parameters, labour distribution and growth

$L^{**}_{R\&D}$	β							
	0,90	**0,91**	**0,92**	**0,93**	**0,94**	**0,95**	**0,96**	**0,97**
ϕ								
1,67	0,75	0,72	0,70	0,68	0,66	0,65	0,64	0,62
1,77	0,67	0,65	0,64	0,63	0,61	0,60	0,59	0,59
1,87	0,61	0,60	0,59	0,58	0,57	0,57	0,56	0,55
1,97	0,57	0,56	0,56	0,55	0,54	0,53	0,53	0,52
2,07	0,54	0,53	0,52	0,52	0,51	0,51	0,50	0,50
2,17	0,51	0,50	0,50	0,49	0,49	0,48	0,48	0,47
2,27	0,48	0,48	0,47	0,47	0,46	0,46	0,45	0,45
2,37	0,46	0,45	0,45	0,44	0,44	0,44	0,43	0,43
2,47	0,44	0,43	0,43	0,43	0,42	0,42	0,42	0,41
2,57	0,42	0,41	0,41	0,41	0,40	0,40	0,40	0,40
2,67	0,40	0,40	0,39	0,39	0,39	0,39	0,38	0,38
g^{**}	β							
	0,90	**0,91**	**0,92**	**0,93**	**0,94**	**0,95**	**0,96**	**0,97**
ϕ								
1,67	0,26	0,20	0,16	0,13	0,10	0,08	0,06	0,04
1,77	0,18	0,15	0,13	0,11	0,09	0,07	0,05	0,04
1,87	0,15	0,13	0,11	0,09	0,07	0,06	0,05	0,03
1,97	0,13	0,11	0,09	0,08	0,07	0,05	0,04	0,03
2,07	0,11	0,10	0,08	0,07	0,06	0,05	0,04	0,03
2,17	0,10	0,09	0,08	0,06	0,05	0,04	0,03	0,02
2,27	0,09	0,08	0,07	0,06	0,05	0,04	0,03	0,02
2,37	0,08	0,07	0,06	0,05	0,04	0,04	0,03	0,02
2,47	0,08	0,07	0,06	0,05	0,04	0,03	0,03	0,02
2,57	0,07	0,06	0,05	0,05	0,04	0,03	0,02	0,02
2,67	0,07	0,06	0,05	0,04	0,04	0,03	0,02	0,02
α	0,50							

Table 4: Calibration of parameters: Labor distribution and Growth rates

5.3 The Intertemporal Elasticity of Substitution (IES) and the propensity to consume

We commented already that the utility function we have introduced in the model has non-homothetic preferences. Increases in income are not equally distributed over the different goods. (See Garcia-Torres (2009)). This utility function presents the characteristic of creating a new product in every new time interval. The Intertemporal Elasticity of Substitution (IES) is the relative variation in consumption levels related to the variation in relative changes in the utility. To be able to appreciate these changes we shall have to calculate the IES at the product level, and add it using the expenditures function.

To demonstrate that the marginal propensity to consume increases we shall calculate the IES in the short run (IES_{sr}) and also in the very long run when t goes to infinity (IES_{lr}). Three hypothetical cases are possible:

- Case 1: $IES_{sr} = IES_{lr}$
  Savings are constant over time. As income increases constant proportions are devoted to savings and consumption. In such a case the propensity to consume is also constant over time. This is the classical result of homothetic preference models.
- Case 2: $IES_{sr} < IES_{lr}$
  As incomes grow people save an increasing proportion of their incomes. In this case the marginal propensity to consume decreases with time.
- Case 3: $IES_{sr} > IES_{lr}$
  People save less and less as income grows. In other words they consume an increasing proportion of their incomes. Therefore the marginal propensity to consume increases with time.

5.3.1 The IES in the short run

Our utility function is:

$$(\sum_{i=1}^{t} \gamma^{(i-1)\alpha} x_i^{\alpha}))^{1/\alpha}$$

Let us calculate for a simple case with three goods. The utility function in this case will be:

$$\left(x_1 + \gamma^{\alpha} x_2^{\alpha} + \gamma^{2\alpha} x_3^{\alpha}\right)^{1/\alpha}$$

Here we can calculate the IES for each good, such that:

$$IES_{x1} = -\frac{U'(x_1)}{U''(x_1)x_1} = \frac{1}{1-\alpha}\frac{x_1+\gamma^{\alpha}x_2^{\alpha}+\gamma^{2\alpha}x_3^{\alpha}}{\gamma^{\alpha}x_2^{\alpha}+\gamma^{2\alpha}x_3^{\alpha}} =$$
$$= \frac{1}{1-\alpha}\frac{x_1+\gamma^{\alpha}x_2^{\alpha}+\gamma^{2\alpha}x_3^{\alpha}}{x_1+\gamma^{\alpha}x_2^{\alpha}+\gamma^{2\alpha}x_3^{\alpha}-x_1}$$
$$IES_{x2} = -\frac{U'(x_2)}{U''(x_2)x_2} = \frac{1}{1-\alpha}\frac{x_1+\gamma^{\alpha}x_2^{\alpha}+\gamma^{2\alpha}x_3^{\alpha}}{\gamma^{\alpha}x_2^{\alpha}+\gamma^{2\alpha}x_3^{\alpha}} =$$
$$= \frac{1}{1-\alpha}\frac{x_1+\gamma^{\alpha}x_2^{\alpha}+\gamma^{2\alpha}x_3^{\alpha}}{x_1+\gamma^{\alpha}x_2^{\alpha}+\gamma^{2\alpha}x_3^{\alpha}-\gamma^{\alpha}x_2^{\alpha}}$$
$$IES_{x3} = -\frac{U'(x_3)}{U''(x_3)x_3} = \frac{1}{1-\alpha}\frac{x_1+\gamma^{\alpha}x_2^{\alpha}+\gamma^{2\alpha}x_3^{\alpha}}{x_1+\gamma^{\alpha}x_2^{\alpha}} =$$
$$= \frac{1}{1-\alpha}\frac{x_1+\gamma^{\alpha}x_2^{\alpha}+\gamma^{2\alpha}x_3^{\alpha}}{x_1+\gamma^{\alpha}x_2^{\alpha}+\gamma^{2\alpha}x_3^{\alpha}-\gamma^{2\alpha}x_3^{\alpha}}$$

The general case, in any fixed t, and for any good j, is:

$$IES_{x_j} = \frac{1}{1-\alpha}\frac{\sum_{i=1}^{t}\gamma^{(i-1)\alpha}x_i^\alpha}{\sum_{i=1}^{t}\gamma^{(i-1)\alpha}x_i^\alpha - \gamma^{(j-1)\alpha}x_j^\alpha}$$

Now to study how consumption changes in the short term, still using the example with three goods:

$$IES_{x1}\frac{p_{1t}x_1}{E_t} + IES_{x2}\frac{p_{2t}x_2}{E_t} + IES_{x3}\frac{p_{3t}x_3}{E_t}$$

with

$$E_t = p_{1t}x_1 + p_{2t}x_2 + p_{3t}x_3$$

If markets are in equilibrium prices are equal for all goods in each period. We can insert the static solution for each period, producing an expression for E_t which will be:

$$E_t = p_{1t}^*x_1^* + p_{2t}^*x_2^* + p_{3t}^*x_3^* = p_t(1+\gamma+\gamma^2)$$

$$IES_{x1}\frac{1}{1+\gamma+\gamma^2} + IES_{x2}\frac{\gamma}{1+\gamma+\gamma^2} + \underbrace{IES_{x3}\frac{\gamma^2}{1+\gamma+\gamma^2}}_{\textit{last good}}$$

The last good is the one which gets the highest proportion of incomes, therefore plays the most important role in determining how incomes and savings react as the economy continues to grow.

Let us study the sign of the IES for the last good in t=3:

$$IES_{x_3} = \frac{1}{1-\alpha}\frac{x_1 + \gamma^\alpha x_2^\alpha + \gamma^{2\alpha}x_3^\alpha}{x_1 + \gamma^\alpha x_2^\alpha}$$

Choosing $p_t x_1 = 1$ as the numeraire, the equation has the following form:

$$IES_{x_3} = \frac{1}{1-\alpha}\frac{\gamma + \gamma^{2\alpha} + \gamma^{4\alpha}}{\gamma + \gamma^{2\alpha}}$$

which could be also be written as:

$$IES_{x_3} = \frac{1}{1-\alpha}(1 + \frac{\gamma^{4\alpha}}{\gamma + \gamma^{2\alpha}})$$

This expression dominates the dynamics of the IES in the short run for this case with three goods. The main difficulty of the model is that the last good, the one determining the dominant behaviour, continually changes. However we are able to calculate, when all markets are in equilibrium and after choosing the numeraire, the value of this dominant good:

$$IES_{x_{tt}} = \frac{1}{1-\alpha}(1 + \frac{\gamma^{2t\alpha}}{1 + \gamma^{2\alpha} + ... + \gamma^{2(t-1)\alpha}}) \quad (40)$$

The function will give us the value of the short-term IES, at any time t, for the last good. With a γ greater than one and positive α less than one, this term is always positive and greater than one.

5.3.2 The IES in the long run

If we calculate the limit of expression 40 as t goes to infinity:

$$\lim_{t\to\infty} IES_{x_{tt}} = \lim_{t\to\infty} \frac{1}{1-\alpha}(1+\frac{\gamma^{2t\alpha}}{1+\gamma^{2\alpha}+...+\gamma^{2(t-1)\alpha}}) = \frac{1}{1-\alpha}(\gamma^{2\alpha}) \quad (41)$$

Solution 41 of this limit is the value of the IES in the long run for the dominant good, for any t, and with a γ greater than one and positive α.

$$\underbrace{\frac{1}{1-\alpha}(1+\frac{\gamma^{2t\alpha}}{1+\gamma^{2\alpha}+...+\gamma^{2(t-1)\alpha}})}_{(IES_{sr})} > \underbrace{\frac{1}{1-\alpha}(\gamma^{\alpha 2})}_{(IES_{lr})}$$

This is the third case, when people save less and less as income increases.[11] This suggests that the marginal propensity to consume increases with time. It is a logical result since we have modelled a technological effect that makes people need more goods as innovations arrive.

6 Conclusion

To understand what is happening in this economy let us turn to figure 4. Arrows show how the different actors are related.

Consumers get income from work they do in one of the two sectors as well as profits from the shares they have in both sectors. They get the returns on their savings. With this income they decide how much to save and how much to consume. The savings go to the financial markets; the markets will use this money to grant loans to the R&D and final goods sectors.

Final goods firms get profits from selling the final goods to consumers. Each of them produces a different good for which they have total control over the knowledge involved in its production. To have this right they must buy the patent from the R&D sector. To pay the initial cost they borrow money from the financial markets.

The R&D sector produces a single blueprint in each time interval. To pay the labour cost of generating the next innovation they ask for credit from the financial sector. The financial sector in return has some form of investment in the R&D activity which it distributes equally across all individuals. The

[11] We have compared the value of the dominant goods in the short run and in the long run. It is possible to show that even if we work with the sum of all IES for all existing goods the relation holds in both cases. The limit of the value of each good's IES as t goes to infinity is zero for all goods, except for the two most recent ones. We have shown above the calculation for the most recent good; for its predecessor the short-term IES's limiting value is $\frac{1}{1-\alpha}$. For earlier goods in the chain of goods, in the short run we always have a positive value, while in the long run the limit is zero. Therefore the relation holds even if we work with all the goods simultaneously.

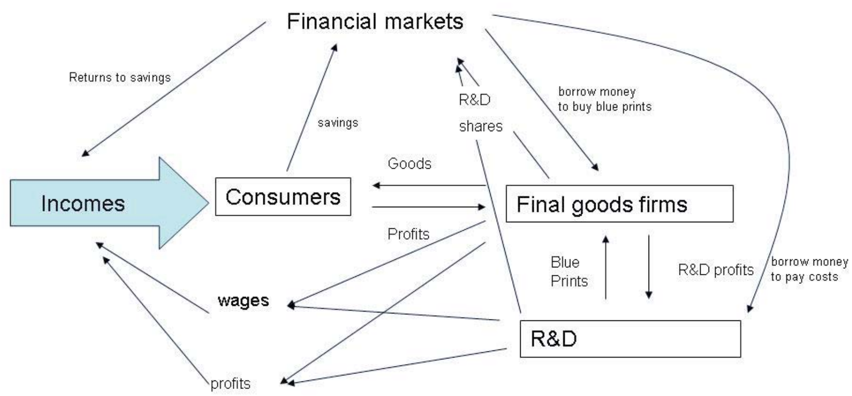

Figure 4: Interaction of agents

value of the innovation is determined by the total labour devoted by the society to development of blueprints.

Each time an innovation arrives there will be an increase in the value of the final goods produced. Knowledge has the effect of creating new blueprints. Because this knowledge is profitable there will be an increase in incomes which will be translated into growth of the society's wealth and of the utility experienced by its consumers.

The main implication of the model is that economic growth is generated based on an active role of demand. The production coefficients are fixed, which means total production is always constant, labour is the only production factor and there is no population growth. The only factor that shows increasing returns to scale is knowledge, through the activities of the R&D sector. Every time period the same amount of people working in the R&D comes up with an idea which has higher value for the consumers than that of all earlier exploited ideas.

At the macroeconomic level two things permitting growth occur. At each time step a worthwhile innovation arrives whose value is recognised by the final goods sector. When this idea is bought money (equal to the value of the patent) is injected in the economy. At the same time the average propensity to consume increases since there is a new good that is sought after by all the economy. Together the two effects allow the economy to present a self sustaining growth process.

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