MERIT-Infonomics Research Memorandum series

Non linear dynamics in US macroeconomic time series

Franco Bevilacqua

2001-035

MERIT – Maastricht Economic Research Institute on Innovation and Technology
PO Box 616
6200 MD Maastricht
The Netherlands
T: +31 43 3883875
F: +31 43 3884905

http://meritbbs.unimaas.nl
e-mail:secr-merit@merit.unimaas.nl

International Institute of Infonomics

PO Box 2606
6401 DC Heerlen
The Netherlands
T: +31 45 5707690
F: +31 45 5706262

http://www.infonomics.nl
e-mail: secr@infonomics.nl

Non linear dynamics in US macroeconomic time series

Franco Bevilacqua

October 2001

Keywords: *i.i.d.* processes, chaos

JEL: C22, E13, E32

Abstract

This paper investigates whether the inherent non stationarity of the US macroeconomic time series may be entirely explained by simple stochastic non linear models (like GARCH).

Applying the numerical tools of the analysis of dynamical systems to long time series for the US, we reject the hypothesis that the uncorrelated and homoscedastic residuals of the estimated GARCH models contain no structure. Contrary to the theories that attribute the source of the irregular behaviour of the economic system to erratic factors, we are not able, using GARCH models, to obtain truly random residuals. Given this evidence we put forward the possibility that seemingly but not truly random residuals could be, in principle, better controlled and forecasted in the short run.

Acknowledgements

Bevilacqua gratefully acknowledges financial support from Compagnia di San Paolo and Fondazione Einaudi Torino.

Franco Bevilacqua

MERIT, Maastricht University, PO Box 616, 6200 MD Maastricht, The Netherlands

Phone: +31-(0)43-3883879, Email: f.bevilacqua@merit.unimaas.nl

1 Introduction[1]

In this paper we check whether the observed fluctuations in macroeconomic time series is solely due to random exogenous shocks which perturb a stable system. While in a previous paper (Bevilacqua and van Zon 2001[2]) it was shown that the uncorrelated residuals of an autoregressive linear AR model do have a structure while they are assumed not to have any, here we show that also in the case we model non linearities in variance by means of a GARCH model we still obtain estimated residuals that look random but nevertheless they contain non linear structure.

These results may suggest that there is still room to leave more about the dynamics of macroeconomic variables and that both linear and non linear autoregressive models (that are based on exogenous plugged in additive noise) are not able to fully explain the whole story when the phenomena we deal with are intrinsically unstable.

This paper is organized in the following way: in section 2 we underline, following the King, Rebelo and Plosser (1988 a[3], b[4]) papers, the intimate link between neoclassical economic theory and autoregressive models; in section 3 we outline how we processed the updated and extended Nelson and Plosser (1982)[5] time series; in section 4 we show that there is unexplained deterministic structure left in the autoregressive models also in the case we use a GARCH model; in section 5 we conclude that, in line with previous results (Bevilacqua and van Zon 2001), that both linear and non linear autoregressive models show not to have random residuals.

2 The link between growth models and AR type models

The research interest of neoclassical growth models of economic fluctuations has been considerable after the Nelson and Plosser contribution. A number of contributions based on the neoclassical model of capital accumulation with additional persistent exogenous shocks have been proposed in the '80s by Kydland and

---

[1] This paper was presented at the conference "2001 European Workshop on General Equilibrium Theory" in Maastricht.

[2] F. Bevilacqua and van Zon A., *Random walks and nonlinear paths in macroeconomic time series*, 2001-007 MERIT-Infonomics 2001 Research Memoranda, Maastricht.

[3] R. G. King, C. I. Plosser and Rebelo S. T., "Production, growth and business cycles I: the basic neoclassical model", *Journal of Monetary Economics*, vol. 21, 1988, pp. 274-308.

[4] R. G. King, C. I. Plosser and Rebelo S. T., "Production, growth and business cycles II: new directions", *Journal of Monetary Economics*, vol. 21, , 1988, pp. 309-41.

[5] C. R. Nelson and Plosser C.I., "Trends and random walks in macroeconomic time series: some evidence and implications", *Journal of Monetary Economics*, vol.10, 1982, pp. 139-62.

Prescott (1982)[6], Long and Plosser (1983)[7], Hansen (1985)[8] and many others. The perfect matching between stable long run neoclassical growth models and AR models, has allowed economists to explain both the economic fluctuations and the irreversible course of history[9].

Contrary to what was taken for granted by Friedman (1963)[10] and the onward research on business cycles[11], the long term effect of each exogenous shock suggest that latter have a real nature rather than a monetary one. Most of *Real* Business Cycle models developed in the '80s and '90s attributed the cause of economic fluctuations to exogenous shocks from changes in technology, preferences, terms of trade and economic policies and *not* to monetary factors.

The existence of non stationarity in economic time series was confirmed by means of the Dickey-Fuller test[12] and this result was consistent with a general equilibrium growth theory which considered random and cumulative total factor productivity shocks. These models, and specifically the neoclassical models, may indeed easily embed random shocks in order to replicate the *apparent* stochastic growth of the macroeconomic time series[13].

Given this empirical evidence, we stress the point that growth only appears to follow a stochastic path because, as we will show in the final part of this

---

[6] F. E. Kydland and Prescott E.C., "Time to build and aggregate fluctuations", *Econometrica*, vol. 50, 1982, pp. 1345-1370.

[7] J.B. Long and Plosser C.I., "Real business cycles", *Journal of Political Economy*, vol. 91, 1983, pp. 39-69.

[8] G. Hansen, "Indivisible labor and business cycle", *Journal of Monetary Economics*, vol. 16, 1985, pp. 309-327.

[9] In Kydland and Prescott (1982), for example, the amplitude of fluctuations generated by means of a total factor productivity random generator, surprisingly mimics the actual dynamics of the post war US time series.

[10] M. Friedman, "The role of monetary policy", *American Economic Review*, vol. 58, 1963, pp. 1-17.

[11] According to E. Prescott, "Notes on Business Cycle Theory: methods and problems", Siena, paper presented at ISER Conference july 1998, Friedman and Schwarz (1963) showed that monetary shocks were the main source of business cycles fluctuations. The Friedman and Schwarz thesis was widely accepted in the '60s and '70s because real growth models seemed to be inconsistent with data, i.e. the monetary shocks could explain short run fluctuations in a long run process of growth that was explained by economic theory.

[12] For many recent related works around the non stationarity of economic time series see: G. S. Maddala and Kim I. M., *Unit roots, cointegration and structural change*, Cambridge, Cambridge University Press, 1998.

[13] The apparent persistency and stochasticity in the dynamics of economic time series seems to go back further the Nelson and Plosser 1982 paper.

R. Lucas in "Understanding Business Cycles" edited by Brunner K., Meltzer, *Stabilization of the domestic and international economy*, Carnegie-Rochester Conference Series on Pubblic Policy, vol. 5, 1977, pp. 7-29. writes:

Technically movements about trend in gross national product in any country can be well described by a stochastically disturbed difference equation of very low order. These movements do not exhibit uniformity of either period or amplitude, which is to say, they do not resemble the deterministic wave motions which sometimes arise in the natural sciences. Those regularities which are observed are in the co-movements among different aggregative time series".

The issue around an apparent random walk dynamics in economic time series is therefore an old one. Nelson and Plosser have provided the statistical proof, via the Dickey-Fuller test, that most of economic time series follow a random walk.

paper, the residuals of fitted autoregressive models incorporate a deterministic (and potentially forecastable) structure that is neglected by this econometric model and by linear economic models.

In conclusion, this paper provides empirical evidence, or at least a partial justification, for non linear economic models from the Dynamic General Equilibrium Theory put forward firstly by Gale (1973)[14], Benhabib and Nishimura (1979[15], 1985[16]), Benhabib and Day (1982)[17], Grandmont (1985)[18], Boldrin and Montrucchio (1986)[19] whose models are characterized by an endogenously generated dynamics, and for the evolutionary models by Freeman, Clark and Soete (1982)[20], Silverberg (1984)[21], Silverberg and Lenhert (1993)[22], Silverberg and Verspagen (1998)[23] in which random exogenous shocks (*mutation* factors) perturb a long run trajectory of a *self-organizing* dynamical system.

2.0.1 Neoclassical and autoregressive models

On the one hand, it is widely accepted that neoclassical growth models (both the old neoclassical models with exogenous technical change and the endogenous neoclassical growth models)[24] are characterized by a constant long run growth rate for most of the macroeconomic variables such as production, consumption, capital and investment per capita.

On the other hand, Nelson and Plosser have shown that the hypothesis that macroeconomic variables evolve as integrated stochastic processes could not generally be rejected. This result invalidated most of the econometric works

[14] D. Gale, "Pure exchange equilibrium of dynamic economic models", *Journal of Economic Theory*, vol. 6, 1973, pp. 12-36.

[15] J. Benhabib and Nishimura K., "The Hopf bifurcation and the existence and stability of closed orbits in multisector models of optimal economic growth", *Journal of Economic Theory*, vol. 21, 1979 pp. 421-444.

[16] J. Benhabib and Nishimura K., "Competitive equilibrium cycles", *Journal of Economic Theory*, vol. 35, 1985, pp. 284-306.

[17] J. Benhabib and Day R. H., "A characterization of erratic dynasmics in overlapping generation model", *Journal of Economic Dynamics and Control*, vol. 4, 1982, pp. 37-55.

[18] J.M. Grandmont, "On endogenous competitive business cycles", *Econometrica*, vol. 53, 1985, pp. 995-1046.

[19] M. Boldrin and Montrucchio L., "On the indeterminacy of capital accumulation paths", *Journal of Economic Theory*, vol. 40, 1986, pp. 26-39.

[20] C. Freeman, J. Clark and Soete L., *Unemployment and technical innovation: a study of long waves in economic development*, London, Pinter 1982.

[21] G. Silverberg, "Embodied technical progress in a dynamic economic model: the self organization paradigm", in R.M. Goodwin et Al., *Nonlinear dynamics in economics*, Cambridge University Press, 1984.

[22] Silverberg G. and Lenhert D., "Long waves and evolutionary chaos in a simple schumpeterian model of embodied technical change", *Structural Change and Economic Dynamics*, vol. 4, 1993, pp. 9-37.

[23] G. Silverberg and Verspagen B., "Economic growth and economic evolution: a modeling perspective", in F. Schweitzer and Silverberg G., *Selbsorganisation. Jahrbuch fur komplexitat in den natur-, sozial- und geisteswissenschaften*, Berlin, Duncker & Humblot, 1998.

[24] See the original papers and King, Rebelo and Plosser 1988a and 1988b.

made until the end of the '70s that were consistent with the Solow model with a deterministic and constant trend with reversible monetary shocks.

However the impasse around the apparent contradiction between economic theory and empirical evidence was readily solved by King, Rebelo and Plosser. They showed that growth theory, which assume steady growth, may be consistent with the highly irregular behavior of economic time series.

They considered the a one-commodity Solow (1956)[25] and Swan (1956)[26] model. The production function, the capital accumulation equation and the resource constraint are:

$$\begin{array}{cc} Y_t = A_t K_t^{1-\alpha} (N X_t)^{\alpha} & 0 < \alpha < 1 \\ K_{t+1} = I + (1-\delta) K_t = s A_t K_t^{1-\alpha} (N X_t)^{\alpha} + (1-\delta) K_t & \\ C_t + I_t = Y_t & \end{array} \quad (1)$$

where Y_t is the output at time t, K_t is the capital stock available at time t and δ its depreciation rate, s the saving rate, N is the labor input that is assumed constant at all time t, A_t is a multiplicative factor and its change corresponds to temporary changes of total factor productivity, $X_t N$ is the labor measured in *efficiency* units and moreover, changes of X_t modify permanently the performance of the system, C_t is the consumption at time t[27].

Assume constant returns to scale in the production function, and *constant* labor augmenting technical change at rate $\frac{\Delta X}{X}$. The dynamic equation for the capital stock may be rewritten as:

$$\begin{array}{c} \Delta K_t = s A_t K_t^{1-\alpha} (N X_t)^{\alpha} - \delta K_t \\ \gamma = \frac{\Delta k_t}{k_t} = \frac{s A_t k_t^{1-\alpha} (X_t)^{\alpha} - \delta k_t}{k_t} = \frac{s A_t k_t^{1-\alpha} (X_t)^{\alpha}}{k_t} - \delta \text{ where } k_t = \frac{K_t}{N} \end{array} \quad (2)$$

where γ is the growth rate of the capital stock k_t per capita.

If $\frac{s A_t k_t^{1-\alpha} (X_t)^{\alpha}}{k_t} > \delta$, $\frac{\Delta k_t}{k_t} > 0$, capital per capita grows.

Conversely, if $\frac{s A_t k_t^{1-\alpha} (X_t)^{\alpha}}{k_t} < \delta$, $\frac{\Delta k_t}{k_t} < 0$, capital stock per capita decreases.

In steady state growth $\frac{\Delta k_t}{k_t}$ is constant, and therefore $\frac{A_t k_t^{1-\alpha} (X_t)^{\alpha}}{k_t}$ is also constant. In order that $A_t k_t^{-\alpha} (X_t)^{\alpha}$ is constant over time, k_t and X_t must grow at the same rate γ. The output per capita is $y_t = A_t k_t^{1-\alpha} (X_t)^{\alpha} = k A_t k_t^{-\alpha} (X_t)^{\alpha}$; in steady state, being $A_t k_t^{-\alpha} (X_t)^{\alpha}$ constant, also y_t grows at the same rate of

[25] R. Solow, "A contribution to the theory of economic growth", *Quarterly Journal of Economics*, vol. 70, 1956, pp. 65-94.

[26] T. W Swan, "Economic growth and capital accumulation", *Economic Record*, vol. 32, 1956, pp. 334-361.

[27] Where the consumption decisions are based on a well behaved utility function $U = \sum_{t=0}^{\infty} \beta^t u(C_t, L_t)$ with $\beta < 1$
where L_t is the leisure at time t, u the utility. ∞ stands to indicate that the individual is the infinite lived representative.

k, γ. Consumption per capita is $c = (1-s)y$ and therefore grows at the same rate γ over time. In this sense, macroeconomic variables follow a (loglinear) deterministic trend.

This view was in sharp contrast with the empirical evidence from Nelson and Plosser (1982) who showed that the existence of a stochastic trend should not be neglected. However it is very easy to make the basic version of the deterministic neoclassical model stochastic.

To do that, we consider that the labor augmenting technical change occurs *stochastically* as a random walk.

We have:

$$X_\tau = X_0 \gamma^\tau e^{\sum_{t=0}^{\tau-1} \varepsilon_{t-i}} \rightarrow \ln X_\tau = \ln X_0 + \tau \ln \gamma + \sum_{t=0}^{\tau-1} \varepsilon_{t-i} \qquad (3)$$

where $\sum_{t=0}^{\tau-1} \varepsilon_{t-i}$ represent permanent shifts of $\ln X_\tau$ which do not average out.

Given the dynamic equation for capital accumulation, in steady state $\frac{\Delta k_\tau}{k_\tau}$ is constant and $\frac{A_\tau k_\tau^{1-\alpha} (X_\tau)^\alpha}{k_\tau} = A_\tau k_\tau^{-\alpha} (X_\tau)^\alpha$ is also constant. In order that $A_\tau k_\tau^{-\alpha} (X_\tau)^\alpha$ is constant over time, k_τ and X_τ must grow at the same stochastically by $\gamma^\tau e^{\sum_{t=0}^{\tau-1} \varepsilon_{t-i}}$:

$$\ln k_\tau = \ln k_0 + \tau \ln \gamma + \sum_{t=0}^{\tau-1} \varepsilon_{t-i} \qquad (4a)$$

which (through (1) and (2)) implies that y_τ and c_τ grow in steady state also by $\gamma^\tau e^{\sum_{t=0}^{\tau-1} \varepsilon_{t-i}}$:

$$\ln y_\tau = \ln y_0 + \tau \ln \gamma + \sum_{t=0}^{\tau-1} \varepsilon_{t-i} \qquad (4b)$$

$$\ln c_\tau = \ln c_0 + \tau \ln \gamma + \sum_{t=0}^{\tau-1} \varepsilon_{t-i} \qquad (4c)$$

in virtue of the fact that $c_t = (1-s)y_t$.

In this sense, macroeconomic variables follow a *stochastic* trend. The above equations may be equivalently rewritten in terms of an AR(1) process where all the economic variables depend on their past value, the average growth rate plus a non transitory stochastic error term. It can be easily shown that equations (4) are the same of equations (5):

$$\ln k_\tau = \ln k_{t-1} + \ln \gamma + \varepsilon_t \qquad (5a)^{28}$$

[28] In fact, $\ln k_\tau = \ln k_0 + \tau \ln \gamma + \sum_{t=0}^{\tau-1} \varepsilon_{t-i} = \ln k_0 + \ln \gamma \sum_{i=0}^{t-1} (1)^i + \sum_{i=0}^{t-1} (1)^i \varepsilon_{t-i} = \ln k_1 + \ln \gamma \sum_{i=0}^{t-2} (1)^i + \sum_{i=0}^{t-2} \varepsilon_{t-i} = ... = \ln k_{t-1} + \ln \gamma + \varepsilon_t$. In the same way, we find equations (5b), (5c) and (5d).

$$\ln y_\tau = \ln y_{t-1} + \ln\gamma + \varepsilon_t \qquad (5b)$$
$$\ln c_\tau = \ln c_{t-1} + \ln\gamma + \varepsilon_t \qquad (5c)$$
$$\ln X_t = \ln X_{t-1} + \ln\gamma + \varepsilon_t \qquad (5d)$$

These results are in line with most of the empirical studies that confirm: 1) macroeconomic variables follow a stochastic trend, i.e. a random walk; 2) macroeconomic variables co-evolve together, i.e. they are cointegrated.

What is striking is that these relationship basically holds for all the time series that were analyzed (Bevilacqua and van Zon 2001) in concordance with the large unit root literature, except for the fact that we found some significant autocorrelation in the residuals[29].

What is implicit in the stochastic version of the neoclassical model, is that the economic system is essentially stable. In fact, if time series follow a random walk and we remove random innovations, we have a stationary stable system. In the absence of technical change the system would never change, except for the occurrence of other exogenous shocks like a change in the preferences for instance.

If the term $\sum_{t=0}^{\tau}\varepsilon_{t-i}$ were not random, what would then be the consequences to economic theory? The first consequence would be that, understanding the deterministic non linear dynamics, we could make a better prediction than simple AR like models, since the best predictor for the residual in the AR models cannot be but its mean value. The second consequence would be that economic systems might be intrinsically unstable, that is also without the injection of exogenous random inputs the system could be not motionless. Given these two conclusions, we would have that real economic time series show to be complex, seemingly random but they contain some deterministic structure; therefore, in principle, they could better forecasted and better controlled via a noise reduction that should *not* be based on a linear filter like autoregressive models.

In the next section we test whether or not the residual component of an optimized autoregressive model is truly random, and we find, to our surprise, that the hypothesis that $\sum_{t=0}^{\tau}\varepsilon_{t-i}$ are truly random is indeed *not* confirmed by our inference, notwithstanding the fact that the residuals in our model possess the same statistical linear properties of white noise.

3 Some notes about the method

Before we proceed on the empirical analysis we summarize the basic steps of our method:

[29] This fact does not represent a real problem since autocorrelation in residuals is easily removed (by means of a *Koyck* transformation) adding lagged regressors (normally two or three), see Johnston J., *Econometric Methods*, Mc Graw Hill, 1984, chap.9.

1) We first selected time series with a certain number of observations[30]. This is because, as Brock et Al. (1991) have pointed out, a number of at least 400 observations is a good starting point, if not a necessary condition, to obtain reliable results from the BDS test[31].

2) We chose and we have relied on seasonally adjusted monthly data to remove trivial non linearities due to seasonality.

3) We take the natural logs of the original time series that grow exponentially over time.

4) We take the first difference with respect to time.

5) We check for stationarity via the augmented Dickey-Fuller test[32].

6) We build our autoregressive model removing any linear autocorrelation in the residuals via Koyck transformation.

7) We check for heteroscedasticity and if this is present we model it to obtain the error terms with zero autocorrelation and zero heteroscedasticity. Modeling heteroscedasticity means to estimate autoregressive models with conditional heteroscedasticity (ARCH-GARCH) models. ARCH-GARCH models are indeed non linear but this class of non linear models resembles the linear AR-ARMA models except for the modelled non linearity in the residuals. What is really important, however, is that, contrary to AR-ARMA models corrected for heteroscedasticity, ARCH-GARCH models have error terms that are also homoscedastic and not just uncorrelated. Because the BDS test detects the presence of non linear structures, and therefore also ARCH structures (which are non linear), we wanted to remove the presence of this type of non linearity in order to search for other structures (different from ARCH) in the residuals.

8) We calculate the values of the maximal Liapunov exponents that characterize the error terms of the estimated model, to see how fast nearby trajectories of the error terms diverge over time. A positive Liapunov exponent is evidence of chaos or noise.

9) We use *Ruelle plots*[33] to uncover, from the qualitative point of view hidden structures in the time series.

10) We test, using the BDS statistic, whether the error terms are independently and identically distributed.

---

[30] The time series we have used are those of the US. The data are provided by the Bureau of Labor and Statistics and the Federal Reserve.

Links to the files concerning monthly seasonally adjusted and in real terms for industry productions were found at: http://www.bog.frb.fed.us/releases/G17/download2.htm

Indexes of industrial production go back to 1919 and the respective base year is 1992.

A table showing the historical consumer price index for all urban consumers beginning from 1913 was available from the BLS at: ftp://ftp.bls.gov/pub/special.requests/cpi/cpiai.txt.

This table refers to all urban consumers with 1982 as the base year.

The seasonally adjusted "hourly wages" time series in this paper refers to the industry of manufacturing and data type "average hourly earnings of production workers".

[31] We exclude the possibility to analyze any time series of GDP and GNP because of the lack of data, since these time series are at most quarterly. The GDP and GNP quarterly data only extend back to 1959. Early in 2000, the Bureau of Economic Analysis has extended these data back to 1929. Links to their data can be found at: http://www.bea.doc.gov/bea/dn1.htm.

[32] See the statistical appendix for some information about this test.

[33] Called also *recurrence plots.*

11) We check our results randomly by shu- ing the time series and we verify whether the results that we obtain from the BDS test applied on a randomly shu- ed error terms are indeed different from the results that we obtained performing the BDS test on the original time series of errors . This verification, known also as *shu- e diagnostic*[34], is extremely important for us since, if the two results turn out to be different, it means that the time order of the original time series is significant and there exists causality in data[35].

4 Empirical evidence: the US time series

In the analysis that follows we focus on some main macroeconomic and sectoral US time series. We check whether it is possible to extract signals from the error terms that economic literature has assumed to be stochastic. What we want to ascertain is whether or not the residuals embody some non linear components. In other words, we are trying to find out whether important temporal linkages are present in the residuals.

4.1 Industrial production

The time series for industrial production is certainly one of the most complete available. Data go back to 1919 and the frequency of observation is monthly.

In Bevilacqua and van Zon (2001), applying the Dickey-Fuller test[36] to the log of the observed values, we could not reject the null hypothesis of a unit root

[34] See H. W. Lorentz, *Lecture notes in economics and mathematical systems*, Berlin, Springer Verlag, 1989. The shu- e diagnostic has been performed via "surrogate time series" following the procedure suggested in H. Kantz and Schreiber T., *Nonlinear time series analysis*, Cambridge, Cambridge University Press, 1997. A "surrogate" time series is essentially the shu- e of the original time series preserving all the linear properties of the time series like frequencies, amplitudes and eventual linear autocorrelations. We have derived the surrogate time series for all the economic time series we have analyzed, but we called them with the more general and less specialistic term of "shu- ed time series".

[35] In other words we try to falsify the results of rejection of the null *i.i.d.* hypothesis. We will proceed to a random shu- e of the time series in order to break any temporal link among data and we will apply non linear dynamics tools on the shu- ed time series. If the results of non linear test on both the original and the shu- ed time series are similar, it means that time linkages are not important and the time series is generated by a stochastic process, otherwise there is evidence that time cannot be ruled out and there exists a nonlinear component.

[36] Since some time series were autocorrelated in the residuals, we have used for all the real time series the "augmented" form of the Dickey-Fuller test including more lags, trend and intercept. The number of lags we have considered is the minimum that is consistent with uncorrelated residuals. See statistical appendix or R. Harris, *Using cointegration analysis in econometric modelling*, London, Prentice Hall, 1995.

(see also Tab. 1). Afterwards we have estimated the following linear model that best fitted the data[37]:

$$Y_t = 0.02 + 0.99Y_{t-1} + 0.51(Y_{t-1} - Y_{t-2}) + (2.89E - 0.5)t + \varepsilon_t$$

where $Y(t)$ was the observed values of the industrial production in terms of real value. The Durbin Watson statistic was 1.95 within the acceptance range 1.89-2.10 and this indicates that the residuals were not serially correlated.

Hence we focused on the estimated residuals which turned out to be affected by heteroscedasticity[38]. Since the BDS test detects GARCH structures[39] we first wanted to remove this odd non linear structure by modeling it via a GARCH model.

The estimated GARCH model is[40]:

$$\begin{gathered} Y_t = 1.00Y_{t-1} + 0.30(Y_{t-1} - Y_{t-2}) + \varepsilon_t \\ \sigma_t^2 = 0.01 + 0.69\sigma_{t-1}^2 - 0.31\varepsilon_{t-1}^2 \end{gathered}$$

Where σ_t^2 is the today's forecasted variance as a function of the forecasted variance and the squared residuals of the past period. We recalculated the Durbin Watson statistic for the exogenous inputs, and it turned out to be 1.94, again within the acceptance range 1.89-2.10. The estimated residual turned out to be homoscedastic too (see the ARCH Test on residuals in Tab. 1 and the statistics appendix) since we cannot reject the null hypothesis of homoscedasticity for high significance levels (well above 5%). In Fig. 1, the estimated residual, i.e. the difference between the fitted model and the actual data, is shown in a graph, while Fig. 2 contains some descriptive information concerning the distribution of these estimated uncorrelated and homoscedastic residuals.

The calculation of the maximal Liapunov exponent depends on the parameter of the embedding dimension m. There exists a maximal Liapunov exponent for each value of m. The maximal Liapunov exponents are all positive for different values of m and this indicates a high sensitivity of the time series with respect to its initial conditions (Tab. 11). However it is worthwhile to note that the calculated maximal Liapunov exponent for this time series is quite lower than the one calculated for a uniform *i.i.d.* process, but remarkably similar to

[37] Note that the t-statistic we have computed recursively in all the tables of this paper, is used to test the hypothesis that a coefficient is equal to "zero".

The corresponding probability is the p-value. If the p-value is lower than its significance, this is taken as evidence to reject the null hypothesis of a zero coefficient.

[38] We have used the Lagrange multiplier ARCH Test for autoregressive conditional heteroskedasticity in the residuals (R. F. Engle, "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation", *Econometrica*, vol. 50, 1982, pp. 987-1008.), see the statistical appendix.

[39] See statistical appendix.

[40] See also Tab. 1.

the Liapunov exponents calculated for a *i.i.d.* process. This shows that the estimated residuals of our GARCH model have values of the maximal Liapunov exponent very much similar to that of a *i.i.d.* process.

However the doubt that some structured dynamics could be present arises from a look at the Ruelle plot (Fig. 3). At first glance comparing Fig. 3 with Fig. 25 (typical of a noise process) the difference between the two pictures is striking. The absence of any continuous diagonal lines is due to the fact that the linear autocorrelations have been successfully removed. However, the presence of continuous vertical and horizontal lines is clear, while conversely noise is characterized by a completely unstructured plot.

To ascertain whether or not the time series is generated by an *i.i.d.* process we have applied the BDS test. The null *i.i.d.* hypothesis is strongly rejected (Tab. 2, column $W_{m,N}$). A similar test based on the same statistic of the BDS test is the dimension test (Tab. 2, column d_m)[41]. The correlation dimension d_m grows very slowly with m. This is typical of a process that is not guided by chance[42].

If we randomize the order of the events of the original time series, we find that the values of the BDS test and the correlation dimension turn out to be very different from the values obtained using the original time series and we correctly always accept the null *i.i.d.* hypothesis for the shu- ed time series (Tab. 3). This shows that the time order of the residuals of the original time series is not random, and that a *temporal causality* in the residuals exists.

We conclude that estimated residuals in industrial production show a structure that cannot come from a stochastic process and a non linear explanation might be necessary to understand the temporal causality of events.

4.2 Empirical analysis of other macroeconomic time series: industrial production in the main US sectors, employment, hourly wages and consumer price index

We have limited our analysis to data of the main sectors of the American economy[43], employment, hourly wages[44] and the consumer price index. Where relevant the economic variables were seasonally adjusted. Again, we have monthly

[41] Without going into the details, the dimension test is based on the fact that a truly stochastic process is characterized by the growth of the correlation dimension with the increase of the embedding dimension; conversely a truly chaotic process is characterized by the correlation dimension that settles down to a constant value when the embedding dimension increases See C. Hommes, *Nonlinear economic dynamics*, Amsterdam, University van Amsterdam, 1998.

[42] Similar results were also obtained adding a small percentage of noise (5% of the variance). We added noise to the time series simply because, when the nonlinear structure is well defined, adding a small stochastic component should not significantly change the result of the test. Even if there were small *i.i.d.* measurement errors these should not call into question the obtained results.

[43] In real value.

[44] In real value.

observations. Data go back to 1947 for the transportation sector, industrial machinery and electrical machinery, 1967 for the hybrid Hi-tech sector (computers, semiconductors and communications), 1939 for employment, 1932 for hourly wages and 1913 for the consumer price index.

Most of the (log transformed) time series, except employment, seem characterized by a unit root, since for most of them we are not able to reject the null *i.i.d.* hypothesis of the Dickey-Fuller test with high confidence levels (higher than 5%) (Tab. 4, 5, 6, 7, 8, 9 and 10)[45]. These results are qualitatively similar to those obtained by Nelson and Plosser. Also the estimated GARCH model contains a unit root: this is readily apparent if we look at the coefficient of the first lagged variable[46]. For all the time series, the estimated residuals of the GARCH model that fits the data best turn out to be serially uncorrelated and homoscedastic. In fact, the null hypothesis of the Durbin-Watson test as well as the ARCH test are never rejected, even at high confidence level for all the time series (see the high p-values of the ARCH test on residuals in Tab. 4, 5, 6, 7, 8, 9 and 10). Our GARCH models seems to fit the actual data very well, and more importantly the estimated residuals are uncorrelated, homoscedastic and they are well behaved in the sense they are symmetrically distributed (see from Fig. 4 to Fig.17). These estimated non linear models seem to depict the actual dynamics well and also we find that the estimated residuals are distributed as white noise. For this reason, the best predictor of the residuals is simply its mean value.

Let us now turn to the other information we have extracted using non linear dynamics tools.

For all the time series we found positive values of the corresponding maximal Liapunov exponents (Tab. 11) and this result suggests that nearby trajectories diverge over time at a positive exponential rate. However on the basis of just the Liapunov exponent we still cannot establish whether or not the time series is a stochastic process or a chaotic one, because a positive Liapunov exponent may be due to either a stochastic process or chaos. The Liapunov exponent measures should therefore *not* be taken as a test for chaos.

Fortunately some convincing qualitative information comes from the Ruelle plots: the presence of structures different from those typical of a noise process is clear from a visual inspection of the recurrence plots. If we compare Fig. 18, 19, 20, 21, 22, 23, 24 with Fig. 25 (Fig. 25 is typical of an unstructured random process), we observe the existence of structures (repetitive continuous lines over time) in the distances (represented by the intensity of grey) between the embedded vectors (represented by each single point in the coordinates) [47].

[45] However for transportation equipment production and industrial machinery production we were not able to reject the null hypothesis only at 1% significance level.

[46] See the estimated equations directly inside from Table 4 to Table 10.

[47] The presence of continuous lines in the recurrence plots indicates that the embedded vectors represented by each point keep approximately the same distance with respect to all the vectors that belong to the continuous line. In a normal *i.i.d.* process, each vector is randomly distant from any other vector and the probability that nearby vectors have similar distances is very low. Thus in a normal *i.i.d.* process we should not notice any continuous line in the recurrence plots.

As a statistical test regarding the presence of non linearities that are not captured from the GARCH model, we use the BDS test. Applying the BDS test to all the time series at our disposal, we are not able to accept the null *i.i.d.* hypothesis. All the series are characterized by high values of the BDS statistic above their respective critical values[48] (column $W_{m,N}$ in Tab. 12, 13, 14, 15, 16, 17, 18[49]). The dimension test[50], based as the BDS test on the calculated value of the correlation dimension, allows us in some cases to measure the dimension of the chaotic attractor that characterizes the time series. This constant value represents the dimension of the chaotic attractor. In all the series we have analyzed, the correlation dimension (column d_m in Tab. 12, 13, 14, 15, 16, 17, 18) grows less than proportionally with respect to "m", but in many cases we cannot detect a clear tendency of the correlation dimension to settle clearly to a constant value. We cannot therefore provide an exact estimate for the dimension of the underlying chaotic attractor[51].

To check whether these results were spurious, we have randomly ordered the real time series and applied BDS and calculated the dimension correlation of the shu- ed time series to see whether temporal linkages were relevant. In all the cases the values of the BDS and the dimension tests of the shu- ed time series were notably different. We could not reject the null hypothesis of the BDS test for all the shu- ed time series and the correlation dimension also was higher (Tab. 3, 19, 20, 21, 22, 23, 24, 25) with respect to the original time series (Tab. 2, 12, 13, 14, 15, 16, 17, 18). This is a confirmation that temporal linkages between residuals are truly important and therefore that just a probabilistic hypothesis on the residuals of macroeconomic time series does not have an empirical foundation.

5 Concluding remarks

We have first shown empirical evidence that seemingly random estimated residuals from a GARCH fit are not truly random. What is certain is that these residuals contain some non linear structure that a simple non linear model like GARCH is not able to capture.

[48] See the Statistical appendix for critical values, power of the test etc.

[49] See also the statistical appendix for the finite sample characteristics of the test.

[50] Note that the "dimension test", contrary to the BDS test, is not really a statistical test since critical values are not specified. It is a numerical tool that suggests the existence a deterministic dynamics when the calculated correlation dimension tend to a fixed value when the embedding dimension grows.

[51] This phenomenon may be due to the presence of a stochastic component in the time series. It is therefore important to filter our data in order to separately analyze the deterministic component and to quantify the dimension of the chaotic attractor. The future application of filters that allow us to reduce and hopefully remove the stochastic component may also allow us to detect the dimension of chaos for all the real time series for which we have already uncovered the presence of chaos.

In Bevilacqua and van Zon (2001), it was shown that the models with a deterministic (linear or broken) or stochastic trend, are based on the hypothesis of *i.i.d.* residuals. There we found that these residuals contained non linearities. However, we also found that residuals were affected by heteroscedasticity. Any correction of an ARMA model for heteroscedasticity improved the estimation of the coefficients of the model, but the heteroscedasticity in the residuals could not be removed.

In this paper we have tried to capture the non linearity by modeling the variance. The obtained estimated residuals from a GARCH model turned out to be homoscedastic, so the non linearity arising from heteroscedasticity was successfully removed.

We have used the BDS test (which has power against ARCH-GARCH structures) on the residuals of the GARCH model to detect eventual non linearities that could not be captured by the GARCH model. We concluded that, for all the main time series of the US economy, estimated residuals show temporal causality.

Statistical appendix

Our basic statistical issue is to understand whether the dynamics behind the residuals is the result of non linearities or just of a random process. In sections 4 and 5 we have analyzed some cases of both artificial and real time series, and we applied to these time series some statistical tools like the Dickey-Fuller unit root test to check for stationarity and the Durbin-Watson statistic to check whether the residuals were serially uncorrelated. This appendix gives some basic information about the statistical tools used in this paper. More technical information about testing for unit roots may be found in Harris (1995)[52], Boswijk (1996)[53] and Maddala and Kim (1998). We also provide some introduction for testing non linear dynamics with the BDS test and measures about the stability of the systems with Liapunov exponents, entropies and the visual tool of recurrence plots. A comprehensive and technical description of the BDS test is found in Brock et Al. (1991)[54], while advanced material about Liapunov exponents, entropies and recurrence plots may be found in Tong (1990)[55] and in Kantz and Schreiber (1997). In this appendix we summarize the logics and some results of both the linear and non linear time series methods that are strictly necessary for the understanding of the paper.

a) The Durbin-Watson test

[52] Harris R., *Using cointegration analysis in econometric modelling*, Prentice Hall, London, 1995.

[53] H. P. Boswijk, *Unit roots and cointegration*, Amsterdam, University van Amsterdam, 1996.

[54] W. A. Brock, D.A Hsieh and LeBaron B., *Nonlinear dynamics, chaos, and instability: Statistical theory and economic evidence*, Cambridge, MIT press, 1991.

[55] H. Tong, *Non-linear time series: a dynamical system approach*, Oxford, Clarendon Press, 1990.

The Durbin-Watson test is a parametric hypothesis test. The Durbin-Watson statistic measures the relation between adjacent residuals. Serial correlation in the residuals that are adjacent in time constitutes a problem that should be removed. In fact, serial correlation leads ordinary least squares to biased estimates of the parameter coefficients, and is symptomatic of bad model specification (Johnston 1984)[56], that is the functional form of the model ($x_t = x_{t-1} + \varepsilon_t$) is inappropriate because some variables (e.g. lagged errors $\varepsilon_t = \rho\varepsilon_{t-1} + v_t$ with $-1 < \rho < 1$ and $v \sim N\left(0, \sigma^2\right)$) are omitted (Harvey 1990)[57].

The Durbin-Watson statistic DW is defined as $DW = \frac{\sum_{t=2}^{T}\left(\hat{\varepsilon}_t - \hat{\varepsilon}_{t-1}\right)^2}{\sum_{t=1}^{T}\hat{\varepsilon}_t^2} \approx 2(1-\rho)$ where $\hat{\varepsilon}_t$ are the OLS estimated residuals. If there is no correlation between adjacent residuals, DW will be around 2. Given the equation $\varepsilon_t = \rho\varepsilon_{t-1} + v_t$ with $v_t \backsim N(0, \sigma)$, the null hypothesis of zero autocorrelation is $H_0 : \rho = 0$,while the alternative is $\rho \neq 0$. Since $DW \approx 2(1-\rho)$, the DW will be close to 2 under the null hypothesis $\rho = 0$. In the case of strong positive serial correlation, it will be near zero. In the case of negative serial correlation, the Durbin-Watson statistic has a value between 2 and 4. Critical values depend on the sample size. In presence of large samples (i.e. more than 200 observations) DW is approximately normally distributed with mean *2* and variance *4/N* with N the number of observations (Harvey 1990). Based on this result, we can easily derive the critical values for any size of the test for one tailed test against either positive or negative autocorrelation. The null hypothesis of zero autocorrelation is rejected if the DW statistic is less than its critical value for the case of positive autocorrelation alternative hypothesis and greater than its critical value for the case of negative autocorrelation alternative hypothesis.

In the case of normal distribution $N\left(2, \frac{4}{N}\right)$ the repartition function $F(z)$ is equal to $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-\frac{u^2}{2}}du$ with $z = \frac{(x-2)}{\sqrt{\frac{4}{N}}}$ and $\frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}}du \sim N(0,1)$.

The null hypothesis of zero autocorrelation is rejected if the DW statistic is less than its critical value for the case of positive autocorrelation alternative hypothesis. The probability that $N\left(2, \frac{4}{N}\right)$ assumes values less than its critical value x is $F(z)$. If the size of the test is 5% we have: $\alpha = P(Z < z) = F(z) = 5\% = 1 - 95\% = 1 - F(1.6449) = F(-1.6449)$. $z = \frac{(x-2)}{\sqrt{\frac{4}{N}}} = -1.6449 \rightarrow x = 2 - 1.6449 * \sqrt{\frac{4}{N}}$.

If the size of the test is 3%, $x = 2 - 1.96 * \sqrt{\frac{4}{N}}$ while for $\alpha = 1\%$, the critical value is $x = 2 - 2.326 * \sqrt{\frac{4}{N}}$

e.g. if N=1021 the critical value corresponding to a 5%, 3% and 1% size of the test are respectively 1.897, 1.877 and 1.854.

[56] J. Johnston, *Econometric Methods*, New York, Mc Graw Hill, 1984.
[57] A. C. Harvey, *The econometric analysis of time series*, Philip Allan, 1990.

As the sample size grows the critical values tend to 2! The null hypothesis of no serial correlation is rejected in favor of *positive* serial correlation if DW is less than its critical value at a fixed level of significance. Similarly the null hypothesis of no serial correlation is rejected in favor of *negative* serial correlation if DW is greater than its critical value at a fixed level of significance.

For a 5% size of the test we have: $\alpha = P(Z > z) = 1 - F(z) = 5\% = 1 - 95\%$, $\rightarrow F(z) = 95\% = F(1.6449)$ and $z = \frac{(x-2)}{\sqrt{\frac{4}{N}}} = 1.6449 \rightarrow x = 2 + 1.6449 * \sqrt{\frac{4}{N}}$.

If the size of the test is 3%, $x = 2 + 1.96 * \sqrt{\frac{4}{N}}$ while for $\alpha = 1\%$, the critical value is $x = 2 + 2.326 * \frac{4}{N}$.

e.g. if N=1021 the critical value corresponding to a 5%, 3% and 1% size of the test are respectively 2.103, 2.123, 2.146.

The rule of thumb suggested by some econometric software of considering serial correlation in serious consideration only for values less 1.5 or greater than 2.5 is therefore wrong for large sample, while it could be accepted for small samples (like 20 or 30 observations). As large sample increases for the same size of the test we should calculate its critical values in the way as it has been shown.

b) The ARCH Test

We have used the Lagrange Multiplier $ARCH$ Test (LM ARCH test) for autoregressive conditional heteroscedasticity in the residuals (Engle 1982[58]).

To test the null hypothesis that there is no $ARCH$ up to a certain order in the residuals, the following regression for the squared residuals is fitted:

$\varepsilon_t^2 = \theta_0 + \theta_1 \varepsilon_{t-1}^2 + ... + u_t$

where ε_t is the residual and u_t an exogenous input. We have used EViews 3.1 software which reports two test statistics from this test regression. The Obs*R-squared statistic is the Engle LM test statistic. The F-statistic is an omitted variable test for the joint significance of all lagged squared residuals. We reject the null hypothesis of zero heteroscedasticity and no omitted variables if the respective p-values are lower than the significance level (generally set at 1, 3, or 5 %).

c) The Dickey-Fuller test

The Dickey Fuller test is a parametric hypothesis test. With the Dickey-Fuller test we are concerned with testing whether the parameter ϕ of the regression equation $x_t = \phi x_{t-1} + \varepsilon_t$ is equal to 1 with $\varepsilon_t \sim i.i.d.(0, \sigma^2)$. x_i with $i = 1...T$ are the natural logarithms of the real quantities. Since real time series do not show an ever increasing growth rate we are only concerned whether

[58] ENGLE R. F., "Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation", *Econometrica*, Vol. 50, 1982, pp. 987-1007.

$\phi = 1$ (i.e. the series is non stationary) or alternatively $\phi < 1$ (i.e. the series is stationary).

Applying the difference operator Δ: $\Delta x_t = x_t - x_{t-1}$, $x_t = \phi x_{t-1} + \varepsilon_t \rightarrow \Delta x_t = (\phi - 1)\, x_{t-1} + \varepsilon_t$. The null hypothesis $\phi = 1$ is equivalent to $(\phi - 1) = 0$ and the alternative to $(\phi - 1) < 0$.

Dickey and Fuller (1976)[59], via Monte Carlo techniques, derived a t-test from the data generated by the random walk process $\Delta x_t = x_{t-1} + \varepsilon_t$. The critical values of the Dickey-Fuller test for prefixed levels (10, 5, 2.5 and 1%) of significance are:

T	10%	5%	2.5%	1%
25	-1.60	-1.95	-2.26	-2.66
50	-1.61	-1.95	-2.25	-2.62
100	-1.61	-1.95	-2.24	-2.60
250	-1.62	-1.95	-2.23	-2.58
500	-1.62	-1.95	-2.23	-2.58
∞	-1.62	-1.95	-2.23	-2.58

The null hypothesis is rejected when the t-ratio is smaller than its critical value. Testing for a unit root using the regression equation $\Delta x_t = (\phi - 1)\, x_{t-1} + \varepsilon_t$ implies that the process has zero mean (i.e. no stochastic trend) and no deterministic trend.

A more general regression equation is: $x_t = \alpha + \beta t + \phi x_{t-1} + \varepsilon_t \rightarrow \Delta x_t = \alpha + \beta t + (\phi - 1)\, x_{t-1} + \varepsilon_t$ where α and β are parameters. α indicates that there is a stochastic trend (drift) while βt indicates that there is a deterministic trend. Given the regression equation $\Delta x_t = \alpha + \beta t + (\phi - 1)\, x_{t-1} + \varepsilon_t$ the critical values of the Dickey-Fuller test are:

T	10%	5%	2.5%	1%
25	-3.24	-3.60	-3.95	-4.38
50	-3.18	-3.50	-3.80	-4.15
100	-3.15	-3.45	-3.73	-4.04
250	-3.13	-3.43	-3.69	-3.99
500	-3.13	-3.42	-3.68	-3.98
∞	-3.12	-3.41	-3.66	-3.96

and the null hypothesis is rejected when the t-ratio is smaller than its critical value. In the case where the data generating process is unknown, the use of the regression equation $\Delta x_t = \alpha + \beta t + (\phi - 1)\, x_{t-1} + \varepsilon_t$ is to be preferred to $\Delta x_t = (\phi - 1)\, x_{t-1} + \varepsilon$ since the latter is only valid when the mean of the time series is zero while we do not know the true mean of the time series. The more general specification of the regression equation prevents us to get spurious results when there is not any a priori information about the existence of a deterministic or stochastic trend in the time series. However $\Delta x_t = \alpha + \beta t + (\phi - 1)\, x_{t-1} + \varepsilon_t$

[59] See Harris 1995.

is an autoregressive model of order 1. If the true data generating process were of order>1, that is Δx_t would depend on other lagged terms than x_{t-1}, ε_t would turn out to be autocorrelated as an effect of the mispecification. Autocorrelated errors invalidate the use of the Dickey-Fuller distribution, which are based on the assumption of white noise. Changing the estimating equation to the *augmented Dickey-Fuller* regression, we have:

$\Delta x_t = \alpha + \beta t + (\phi - 1) x_{t-1} + \sum_{i=1}^{p-1} (\phi_i - 1) \Delta x_{t-1} + \varepsilon_t$ where $(\phi - 1) = \sum_{i=1}^{p} (\phi_i - 1) - 1$

If $(\phi - 1) = 0$, against the alternative $(\phi - 1) < 0, x_t$ contains a unit root. The same critical values of the case $\Delta x_t = \alpha + \beta t + (\phi - 1) x_{t-1} + \varepsilon_t$ may be used, although they are valid as an asymptotic approximation (Boswijk 1996). A large negative t-statistic rejects the hypothesis of a unit root and suggests that the series is stationary. In this paper we have used the augmented form of the Dickey-Fuller test and the critical values are those from Mac Kinnon (1991) for various sample size.

d) Grassberger-Procaccia correlation sum (integral)

The Grassberger-Procaccia correlation sum (1983)[60] is defined as the fraction of all possible pairs of points in a m-dimensional (i.e. vectors of m-elements) lying within a distance ϵ (Dechert 1994, Hommes 1998). Intuitively the correlation sum is a measure of *concentration* of scattered points.

Its formula is:

$C_{m,N}(\epsilon) = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \chi(\epsilon - \|\mathbf{x}_i - \mathbf{x}_j\|)$

where N is the number of observation of a m-dimensional vector time series $\mathbf{x}_i = [x_i, x_{i+1}, ..., x_{i+m-1}]$,

x_i the observations $i = 1, 2, ..., N$.

$\|\mathbf{x}_i - \mathbf{x}_j\|$ is the euclidean distance between vectors, i.e.

$\sqrt{(x_i - x_j)^2 + (x_{i+1} - x_{j+1})^2 + ... + (x_{i+m-1} - x_{j+m-1})^2}$

χ is a function that:

$\chi(\epsilon - \|\mathbf{x}_i - \mathbf{x}_j\|) = 1$ if $\|\mathbf{x}_i - \mathbf{x}_j\| < \epsilon$,

$\chi(\epsilon - \|\mathbf{x}_i - \mathbf{x}_j\|) = 0$ if $\|\mathbf{x}_i - \mathbf{x}_j\| \geq \epsilon$.

Two important theorems (we refer the reader to the original references for the proves) are related to the correlation sum, the correlation dimension and the BDS statistic that will be discussed below:

Theorem 1 *as* $N \longrightarrow \infty$, $C_{m,N}(\epsilon) \longrightarrow C_m(\epsilon) = \Pr(\|\mathbf{x}_i - \mathbf{x}_j\| < \epsilon)$ *with probability one (Brock et al. 1991).*

[60] See W. D. Dechert, *The correlation integral and the independence of gaussian and relatec processes*, SSRI W.P. 9412, Madison, University of Wisconsin, 1994.

Therefore for a sufficiently large number of observations the correlation sum measures the probability that two randomly chosen vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ are ϵ-close to each other:

$$C_{m,N}(\epsilon) = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1}^{N} \chi(\epsilon - \|\mathbf{x}_i - \mathbf{x}_j\|) \sim C_m(\epsilon) = \Pr(\|\mathbf{x}_i - \mathbf{x}_j\| < \epsilon)$$

Theorem 2 *if* $\mathbf{x}_i$ *is generated by a stochastic i.i.d. process then* $\lim_{n\to\infty} (C_{m,N}(\epsilon) - C_{1,N}(\epsilon)^m) \to 0$ *with probability 1 (Brock and Dechert1988)*[61].

Therefore for a sufficiently large number of observations $C_{m,N}(\epsilon) \sim C_{1,N}(\epsilon)^m$ if the underlying process is *i.i.d.*.

e) Correlation Dimension

The correlation dimension d_m is defined as: $d_m = \lim_{\epsilon\to 0} \lim_{N\to\infty} \frac{\ln C_{m,N}(\epsilon)}{\ln \epsilon}$ and can be readily obtained once we have computed the correlation sum $C_{m,N}(\epsilon)$.

Let us analyze some limit cases:

$C_{m,N}(\epsilon) = 1$ is defined as the fraction of all possible pairs of points (or vectors) are lying within a small distance ϵ, so it may assume any value between 0 and 1.

Suppose that $C_{m,N}(\epsilon) = 0$, that is there are no pairs of points (or vectors) lying within a small distance ϵ. $d_m = \frac{\ln 0}{0} = \infty$. For a random process the $d_m \to \infty$.

Suppose that $C_{m,N}(\epsilon)$ increases towards 1, that is the fraction of all possible pairs of points (or vectors) are getting inside within a small distance ϵ. $\ln C_{m,N}(\epsilon)$ decreases towards $\ln 1 = 0$ and with it d_m that tends towards 0. This is the case in which all the observations are all lying close each other and all inside a distance ϵ. The phenomenon is completely stable and determined.

Now non linear mathematical systems are able to generate time series where $0 < d_m < \infty$. For pseudo random generator $d_m \longrightarrow \infty$, while for any other system d_m tend to a finite value. For example in the case of the tent map it is easy to calculate that $d_m \longrightarrow 1$; for this case and others see Hsieh (1991)[62]. With the calculus of the correlation dimension seems therefore to be possible to detect determinism. However this experimental procedure is not a statistical test. Brock, Dechert and Scheinkman (1987)[63] have therefore provided a statistical hypothesis test with a null hypothesis of *i.i.d.* against any departure from *i.i.d.*

[61] W. A. Brock and Dechert W. D., "Theorems on distinguishing deterministic from random systems" in W. Barnett et al., *Dynamic econometric modelling*, Cambridge, Cambridge University Press, 1988.

[62] D. A. Hsieh, "Chaos and nonlinear dynamics: application to financial markets", *Journal of Finance*, vol. 46, 1991, pp.1839-1877.

[63] W.A. Brock, W.D. Dechert and Scheinkman J.A., *A test for independence based on the correlation integral*, Madison, University of Wisconsin, 1987

f) The BDS statistic, size and power

The BDS test is a non parametric hypothesis test. Contrary to parametric tests, like the Durbin-Watson and the Dickey Fuller tests, it does not test whether a particular parameter assumes a given value. Indeed it tests whether data are independent and identically distributed.

We have seen that theorem 2) implies that $C_{m,N}(\epsilon) \sim C_{1,N}(\epsilon)^m$ if the underlying process is *i.i.d.*.
Brock et al (1987) have also proved that $\sqrt{N}C_{m,N}(\epsilon) - C_{1,N}(\epsilon)^m$ converges to a normal distribution (one can also compute $C_{m,N}(\epsilon)$ and $C_{1,N}(\epsilon)$ and show the same results):

Theorem 3 *as* $N \longrightarrow \infty$, *if* $\mathbf{x}_i$ *is generated by a stochastic i.i.d. process then,* $\sqrt{N}C_{m,N}(\epsilon) - C_{1,N}(\epsilon)^m \longrightarrow N(0,\sigma)$ *and* $W_{m,N}(\epsilon) = \sqrt{N}\frac{C_{m,N}(\epsilon)-C_{1,N}(\epsilon)^m}{\sigma_{m,N}(\epsilon)}$ *where* $\sigma_{m,N}(\epsilon)$ *is a consistent estimator of the asymptotic standard error of* $[C_{m,N}(\epsilon) - C_{1,N}(\epsilon)^m]$.

$W_{m,N}(\epsilon) = \sqrt{N}\frac{C_{m,N}(\epsilon)-C_{1,N}(\epsilon)^m}{\sigma_{m,N}(\epsilon)}$ is the BDS statistic and converges in distribution to a standard normal $N(0,1)$.

Size

As $N \longrightarrow \infty$ the critical value corresponding to a 10%, 5% and 2% size of the two side test are respectively $|1.64|$, $|1.96|$ and $|2.33|$. The null hypothesis of *i.i.d.* is rejected if the $W_{m,N}(\epsilon)$ is greater than its critical value at a fixed level of significance.

However as any other test that relies on its asymptotic distribution, we need the critical values for the finite sample distribution. Brock et al. (1991) and Hsieh (1991). These values were found via Monte Carlo simulations. They have generated random number samples of different sizes (100, 500 and 1000) and 6 distributions (standard normal, student-t with 3 degrees of freedom, double exponential, chi square with 4 degrees of freedom, uniform and bimodal). They applied the BDS test and repeated this experiment 2000 times (5000 for samples of 100 and 500 data points) for different values of m ($m = 2, m = 5$ and $m = 10$) and ϵ ($\frac{\epsilon}{\sigma} = \frac{1}{4}, \frac{1}{2}, 1, \frac{3}{2}, 2$). If we use a 5%, 2.5% or 1% significance level, we should reject 5%, 2.5% or 1% of the replications. Brock et al. and Hsieh found the size of the test for different critical values ($\pm 1.64, \pm 1.96, \pm 2.33$ which correspond to 5%, 2.5% or 1% size of the standard normal in case of one side test) of the parameters m and ϵ for different finite sample sizes.

These were the main results from Monte Carlo simulations (see Brock et al. 1991 for all the tables of the BDS test):

1) *The finite sample property is quite poor for samples of 100 points.* We report the results from the normal distribution. We can easily see that the BDS test rejects the null hypothesis *i.i.d.* at least 3 times more than it should.

Size of BDS Statistic, Standard Normal

$m = 2, N = 100$	ϵ/σ 0.25	0.50	1.00	1.50	2.00	$N(0,1)$
$\% < -2.33$	28.3	13.1	3.90	3.94	5.62	1.00
$\% < -1.96$	32.1	17.2	8.02	7.1	9.18	2.50
$\% < -1.64$	35.6	21.9	12.3	12.04	13.8	5.00
$\% > 1.64$	27.9	16.5	10.0	9.02	10.1	5.00
$\% > 1.96$	25.2	13.4	6.44	5.66	7.12	2.50
$\% > 2.33$	23.0	10.4	3.78	2.96	4.5	1.00

Using the *i.i.d.* time series generated by the other distribution (especially the uniform and the bimodal) did not change the picture very much. The null hypothesis is spuriously rejected too often when the sample size is small.

If we increase the sample size to 500 and 1000 data points, the BDS distribution becomes more normal and its asymptotic distribution (the standard normal) gives a much better approximation of the finite sample BDS distribution (especially when N=1000). Similar results (see Brock et al.1991) were obtained when the samples were obtained from the other *i.i.d.* processes (student-t etc.)

Size of BDS Statistic, Standard Normal

$m = 2, N = 500$	ϵ/σ 0.25	0.50	1.00	1.50	2.00	$N(0,1)$
$\% < -2.33$	8.62	1.96	1.10	1.28	1.34	1.00
$\% < -1.96$	13.0	4.44	3.04	3.26	3.52	2.50
$\% < -1.64$	17.1	8.24	5.98	6.20	6.78	5.00
$\% > 1.64$	16.9	9.32	6.92	6.04	6.58	5.00
$\% > 1.96$	12.6	5.76	3.76	3.36	3.86	2.50
$\% > 2.33$	8.98	3.42	1.80	1.68	1.88	1.00

$m = 2, N = 1000$	ϵ/σ 0.25	0.50	1.00	1.50	2.00	$N(0,1)$
$\% < -2.33$	4.65	1.40	1.05	0.90	0.80	1.00
$\% < -1.96$	8.95	3.25	2.90	2.45	2.65	2.50
$\% < -1.64$	13.3	6.55	5.60	6.30	6.15	5.00
$\% > 1.64$	9.50	6.20	4.70	4.20	5.50	5.00
$\% > 1.96$	6.30	3.70	2.25	2.40	2.50	2.50
$\% > 2.33$	3.60	1.55	0.90	0.70	0.90	1.00

2) increasing the embedding dimension m the asymptotic distribution may provide a better approximation of the finite sample BDS distribution.

Size of BDS Statistic, Standard Normal

$m=5, N=500$	ϵ/σ 0.25	0.50	1.00	1.50	2.00	$N(0,1)$
$\% < -2.33$	12.8	0.84	0.94	1.16	1.12	1.00
$\% < -1.96$	17.1	2.48	2.24	2.88	2.92	2.50
$\% < -1.64$	21.8	5.58	5.52	5.62	5.86	5.00
$\% > 1.64$	19.7	7.24	5.12	5.20	5.68	5.00
$\% > 1.96$	16.1	4.56	3.10	2.96	3.16	2.50
$\% > 2.33$	12.9	2.84	1.56	1.28	1.6	1.00

$m=5, N=1000$	ϵ/σ 0.25	0.50	1.00	1.50	2.00	$N(0,1)$
$\% < -2.33$	6.05	0.70	0.70	0.85	0.60	1.00
$\% < -1.96$	9.55	2.25	2.30	2.55	2.50	2.50
$\% < -1.64$	13.7	4.60	5.35	5.50	5.40	5.00
$\% > 1.64$	14.4	6.80	5.35	5.75	5.90	5.00
$\% > 1.96$	11.0	4.20	3.10	3.50	3.55	2.50
$\% > 2.33$	7.55	2.25	1.95	1.70	1.60	1.00

However for large values of m the finite sample property gets poor again. The reason is that there may be too few observations. In fact $W_{m,N}(\epsilon) = \sqrt{N}\frac{C_{m,N}(\epsilon)-C_{1,N}(\epsilon)^m}{\sigma_{m,N}(\epsilon)}$ and if we calculate for example $C_{1,N}(0.25)$ we find $C_{1,N}(0.25) = 0.14$. If $m=10$, $C_{1,N}(0.25)^{10} = 2.89255E-09$. If we compute $C_{1,N}(0.25)^{10}$ when we have for 1000 observations $C_{1,N}(0.25)^{10} = 0$, that is the probability to find pairs of 10-dimensional vectors within $\epsilon = 0.25$ is zero. The computed $W_{m,N}(\epsilon)$ becomes large and we spuriously reject the null *i.i.d.* hypothesis. Brock et al. (1991) suggest to keep the maximal value of m around $\frac{N}{200}$. For $\frac{\epsilon}{\sigma} = 0.25$, $N = 1000$ and $m = 10$ we reject 95% the right null hypothesis instead of 1%. If we increase the ratio $\frac{\epsilon}{\sigma}$ to 1 we have very good results and the $N(0,1)$ is a good approximation of the finite sample BDS distribution. This is why by increasing ϵ we also increase the probability to find vectors closer than ϵ. However from the preceding tables increasing $\frac{\epsilon}{\sigma}$ beyond 2.0 is not generally recommended since the size of the BDS tends to be too small compared to the normal distribution (it would spuriously accept too often the null hypothesis). Mostly the choice of $\frac{\epsilon}{\sigma} = 1$ or 1.5 gives a good size of the test. We have computed the BDS test for many different values of $\frac{\epsilon}{\sigma}$ between 2 and 0.25.

Size of BDS Statistic, Standard Normal			
$m=10, N=1000$	ϵ/σ 0.25	ϵ/σ 1.00	$N(0,1)$
$\% < -2.33$	95.0	0.40	1.00
$\% < -1.96$	95.15	1.35	2.50
$\% < -1.64$	95.4	3.85	5.00
$\% > 1.64$	3.75	6.40	5.00
$\% > 1.96$	3.70	3.90	2.50
$\% > 2.33$	3.60	2.00	1.00

Power

The BDS test has asymptotic power against the following specific alternatives:

- first order autoregression $AR(1)$: $x_t = \rho x_{t-1} + \varepsilon_t$, $|\rho| \leqslant 1$ and $\varepsilon_t \sim N(0,1)$
- first order moving average $MA(1)$: $x_t = \rho\varepsilon_{t-1} + \varepsilon_t$, $|\rho| \leqslant 1$ and $\varepsilon_t \sim N(0,1)$
- tent map: $x_t = 2x_{t-1}$ if $x_t < 0.5$ and $x_t = 2 - 2x_{t-1}$ if $x_t > 0.5$
- threshold autoregression $TAR(1)$ (Lim 1980, see Tong 1990) : $x_t = \rho x_{t-1} + \varepsilon_t$, $|\rho| \leqslant 1$ if $x_t \leqslant \overline{x}$ and $x_t = \varrho x_{t-1} + \varepsilon_t$, $|\varrho| \leqslant 1$ if $x_t \geq \overline{x}$ and $\varepsilon_t \sim N(0,1)$
- non linear moving average NMA (Robinson 1977, see Tong 1990): $x_t = \varepsilon_t + \varepsilon_{t-1}\varepsilon_{t-2}$ and $\varepsilon_t \sim N(0,1)$
- autoregressive conditional heteroscedasticity $ARCH$ (Engle 1982): $x_t = z_t\varepsilon_t$ and $z_t^2 = z_0^2 + \rho x_{t-1}^2$ and $\varepsilon_t \sim N(0,\sigma)$, $0 < \rho < 1$, x_t has variance $\frac{z_0^2}{1-\rho}$
- generalized autoregressive conditional heteroscedasticity $GARCH$ (Bollerslev 1986[64]): $x_t = z_t\varepsilon_t$ and $z_t^2 = z_0^2 + \rho x_{t-1}^2 + \varrho z_{t-1}^2$ and $\varepsilon_t \sim N(0,1)$, $0 < \rho + \varrho < 1$, x_t has variance $\frac{z_0^2}{1-\rho-\varrho}$

It rejects the null hypothesis of *i.i.d.* with probability one for $0 \leqslant \frac{\epsilon}{\sigma} \leqslant 2$.

For finite samples Monte Carlo simulations showed that (see Brock et al. 1991 for all the tables of the BDS test):

1) the BDS test has different power against different alternatives. As an extreme case see that for a sample size of 100 data points and a significance level of 1%, the power against a GARCH model is only 14.4%, that is the probability to accept the alternative when this is the true one is only 14.4%. On the contrary in the case the time series is generated by a the tent map the power is maximal.

Power of BDS Statistic		
$m=2, N=100$	ϵ/σ 1.00	
	tent	GARCH
$\% > 1.64$	100%	25.6%
$\% > 1.96$	100%	20.2%
$\% > 2.33$	100%	14.4%

[64] Bollerslev T., "Generalized autoregressive conditional heteroscedasticity", *Journal of Econometrics*, Vol. 31, 1986, pp. 307-27.

2) the BDS test increases its power with the sample size. As an example, notice that for N=1000, in the case of GARCH model, the power of the test increases to beyond 80%

Power of BDS Statistic

$m = 2$	ϵ/σ		
	1.00		
	GARCH $N = 100$	GARCH $N = 500$	GARCH $N = 1000$
$\% > 1.64$	25.6%	67.8%	90.4%
$\% > 1.96$	20.2%	58.9%	85.9%
$\% > 2.33$	14.4%	48.3%	80.3%

3) the BDS test increases its power with the embedding dimension but for a large embedding dimension the power of the test falls:

Power of BDS Statistic

$N = 1000$	ϵ/σ		
	1.00		
	GARCH $m = 2$	GARCH $m = 5$	GARCH $m = 10$
$\% > 1.64$	90.4%	98.9	0%
$\% > 1.96$	85.9%	98.4%	0%
$\% > 2.33$	80.3%	97.2%	0%

4) The BDS test has good power properties against all the alternative considered when the number of data points is 1000. When the data points available are around 500, the BDS shows good power except for the GARCH alternative:

Power of BDS Statistic

$N = 1000$	ϵ/σ						
$m = 5$	1.00						
	tent	$AR(1)$	$MA(1)$	TAR	NMA	$ARCH$	$GARCH$
$\% > 1.64$	100%	100%	100%	98.8%	100%	100%	98.9%
$\% > 1.96$	100%	100%	100%	97.2%	100%	100%	98.4%
$\% > 2.33$	100%	100%	100%	94.5%	100%	100%	97.2%

Power of BDS Statistic

$N = 500$	ϵ/σ						
$m = 5$	1.00						
	tent	$AR(1)$	$MA(1)$	TAR	NMA	$ARCH$	$GARCH$
$\% > 1.64$	100%	100%	99.6%	98.8%	100%	100%	87.4%
$\% > 1.96$	100%	100%	99.3%	97.2%	100%	100%	83.0%
$\% > 2.33$	100%	100%	98.5%	94.5%	99.6%	99.9%	76.6%

5) Comparing the power results of the BDS test over a 500 data points time series to that of other non linear tests, specifically the Tsay test (1986)[65] and the Engle test, the BDS test performs better or similar to these tests. The Engle test performs slightly better than the BDS tests in the case of GARCH structures. However the BDS contrary to the Engle test (which look for non zero autocovariances) is able to detect non linearities independently from the value of autocovariances.

g) Liapunov exponents

The Liapunov exponent quantifies the sensitive dependence on initial conditions (states). Take for example a one dimensional dynamic system $x_t = f(x_{t-1})$ like the tent map:

$$x_t = 2x_{t-1} \text{ for } x_{t-1} < 0.5$$
$$x_t = 2(1 - x_{t-1}) \text{ for } x_{t-1} < 0.5$$

We know that for the tent map, given an initial state x_0, there will correspond one and only one $x_1$between 0 and 1. If the process were uniformly distributed between 0 and 1, x_1 could assume any value between 0 and 1 with the same probability. In the case of the tent map, x_1 has only one specific correspondent x_{t+1} between 0 and 1. This means that the system is dependent on initial conditions.

If we take another possible initial state $x_0 + \epsilon_0$ close to x_0, $f(x_0 + \epsilon_0)$ will be still close to $f(x_0)$, but it will be more distant than $x_0 + \epsilon_0$ from x_0. After some periods the two orbits will appear to be totally uncorrelated. This is because the two orbits are divergent. The system is characterized by *sensitive* dependence because two nearby initial states lead to two different orbits which are divergent.

The Liapunov exponent measures the average rate of divergence of nearby initial states.

After N periods, the distance between the two orbits is (Hommes 1998, Kantz and Schreiber 1997):

$$\left|f^N(x_0 + \epsilon_0) - f^N(x_0)\right| \approx \left|\left(f^N\right)'(x_0)\,\epsilon_0\right|$$

If we denote with ϵ_N the distance at time N between the two orbits we may define the exponential divergence of nearby orbits as:

$$\epsilon_N = \epsilon_0 e^{\lambda N}$$

$\epsilon_N = \epsilon_0$ when $\lambda = 0$, that is the case of a cyclical series or a steady state

$\epsilon_N < \epsilon_0$ when $\lambda < 0$, that is the case of convergent series towards a steady state

$\epsilon_N > \epsilon_0$ when $\lambda > 0$, that is the case of divergent series

[65] R. Tsay, "Nonlinearity tests for time series", *Biometrica*, 1986 pp.461-46.

$$\left|f^N(x_0+\epsilon_0)-f^N(x_0)\right| \approx \left|\left(f^N\right)'(x_0)\,\epsilon_0\right| = \epsilon_0 e^{\lambda N}$$
$$\left|\left(f^N\right)'(x_0)\right| = e^{\lambda N} \longrightarrow \lambda = \tfrac{1}{N}\ln\left|\left(f^N\right)'(x_0)\right|$$

Using the chain rule for $\left(f^N\right)'(x_0)$ and taking the limit $t \to \infty$

$$\lambda = \lim_{N\to\infty} \tfrac{1}{N}\sum_{i=0}^{N-1} \ln\left|f'(f^i(x_0))\right|$$

λ is the Liapunov exponent. Notice that if we have a positive λ, say $\lambda = \ln 2$, after 10 periods

$$e^{10\ln 2} = 2^{10} = 1024 \longrightarrow \left|f^{10}(x_0+\epsilon_0) - f^{10}(x_0)\right| = 1024\epsilon_0$$

that is after 10 periods the distance between $f^{10}(x_0+\epsilon_0)$ and $f^{10}(x_0)$ is on average 1024 times greater than $f^0(x_0+\epsilon_0)$ and $f^0(x_0)$. If the initial true value were $x_0+\epsilon_0$ and we had measured x_0, after 10 periods we would have an amplified error on average 1024 times greater than the initial error. The larger the Liapunov exponent the more difficult the prediction is, and so the Liapunov exponent is a measure of predictability.

When we have a time series, we do not know the true function f, but we have its realizations, that is a time series. We have used the following algorithm by Kantz (1994)[66] to compute numerically the Liapunov exponent, directly from the time series without knowing the true or an estimated function of f.

Suppose to observe a point (or vector) x_j that is very close to x_i. x_j and x_i are the observed values of the underlying function. Take the distance between these two observations $\epsilon_i = x_j - x_i$. ϵ_i grows exponentially with time.

After N periods, the distance between the two points is:

$$|x_{N+j} - x_{N+i}| = |\epsilon_{N+i}| = \epsilon_i e^{\lambda N}.$$
$$\left|\left(f^N\right)'(x_i)\right| = e^{\lambda N}$$
$$\lambda = \tfrac{1}{N}\ln\left|\left(f^N\right)'(x_i)\right|$$

λ is the value of the Liapunov exponent. Since from one single time series can define as many different Liapunov exponents as the number of embedding dimension m, we can restrict ourself to the maximal Liapunov exponent that is the most relevant for our analysis. Numerically one can derive a robust consistent and unbiased estimator for the maximal Liapunov exponent (Kantz and Schreiber 1997). One computes:

$$\Phi = \tfrac{1}{N}\sum_{i=0}^{N-1} \ln\left(\tfrac{1}{|\xi(x_i)|}\sum_{x\in\xi(x_i)} |x_{j+n} - x_{i+n}|\right)$$

[66] See Kantz, Schreiber (1997).

$\xi(x_i)$ is the neighborhood of x_i with radius ϵ_i. $|\xi(x_i)|$ denotes the number of observed values within the neighborhood of x_i. n is the number of iteration. Φ varies with n and its slope gives an estimate of the Liapunov exponent.

h) recurrence plots

A recurrence plot $(\mathbf{x}_i, \mathbf{x}_j)$ is a graphical representation of the euclidean distance $\|\mathbf{x}_i - \mathbf{x}_j\|$ in the correlation integral in two dimensions.

It is easy to produce a recurrence plot via an ordinary excel program or alike for the χ function in the correlation integral:

$\chi(\epsilon - \|\mathbf{x}_i - \mathbf{x}_j\|) = 1$ if $\|\mathbf{x}_i - \mathbf{x}_j\| < \epsilon$ and we give to the correspondent point $(\mathbf{x}_i, \mathbf{x}_j)$ in the recurrence plot the color white

$\chi(\epsilon - \|\mathbf{x}_i - \mathbf{x}_j\|) = 0$ if $\|\mathbf{x}_i - \mathbf{x}_j\| \geq \epsilon$ and we give to the correspondent point $(\mathbf{x}_i, \mathbf{x}_j)$ in the recurrence plot the color black.

When $\|\mathbf{x}_i - \mathbf{x}_j\| = 0$ the correspondent point $(\mathbf{x}_i, \mathbf{x}_j)$ is white. Along the 45^0 line $\mathbf{x}_i \equiv \mathbf{x}_j$ so that the 45^0 line $\mathbf{x}_i\mathbf{x}_j$ is white.

When $\|\mathbf{x}_i - \mathbf{x}_j\|$ is maximal, $(\mathbf{x}_i, \mathbf{x}_j)$ is black.

When $0 < \|\mathbf{x}_i - \mathbf{x}_j\| < 1$, the $(\mathbf{x}_i, \mathbf{x}_j)$ assumes a grey tone proportional to the euclidean distance.

For random signals, the uniform distribution of grey tones over the entire plot is expected. For non linear systems a more structured recurrence plot may be dominant. Any continuous line and zones characterized by the same grey tone in the plot indicates the existence of correlation between pair of the m-dimensional points $(\mathbf{x}_i, \mathbf{x}_j)$ since they maintain a similar euclidean distance.

Tab 1: industrial production				
ADF Test Statistic	-3.054169	1% Critical Value		-3.9724
		5% Critical Value		-3.4167
		10% Critical Value		-3.1304
Model: $Y(t) = 1.00*Y(t-1) + 0.30*(Y(t-1)-Y(t-2))+ \varepsilon_t$ $\sigma_t^2 = 0.01+0.69\sigma_{t-1}^2 + 0.31\varepsilon_{t-1}^2$				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
Y(t-1)	1.001963	0.000431	2327.129	0
Y(t-1)-Y(t-2)	0.303917	0.045422	6.690965	2.22E-11
C	0.01206052	0.00332172	3.63080303	0.00028254
ARCH(1)	0.30757605	0.0668553	4.6006233	4.21E-06
GARCH(1)	0.6967068	0.04809526	14.4859752	0
R-squared	0.999837	Mean dependent var		48.42925
Adjusted R-squared	0.999836	S.D. dependent var		36.42827
S.E. of regression	0.465842	Akaike info criterion		1.110864
Sum squared resid	210.7154	Schwarz criterion		1.145832
Durbin-Watson stat	1.942195	5% Critical Value 3% Critical Value 1% Critical Value	2.10 2.13 2.15	1.89 1.87 1.85
ARCH Test on residuals:	Obs*R-squared 0.39868	Probability 0.527433	F-statistic 0.52792	Probability 0.39868

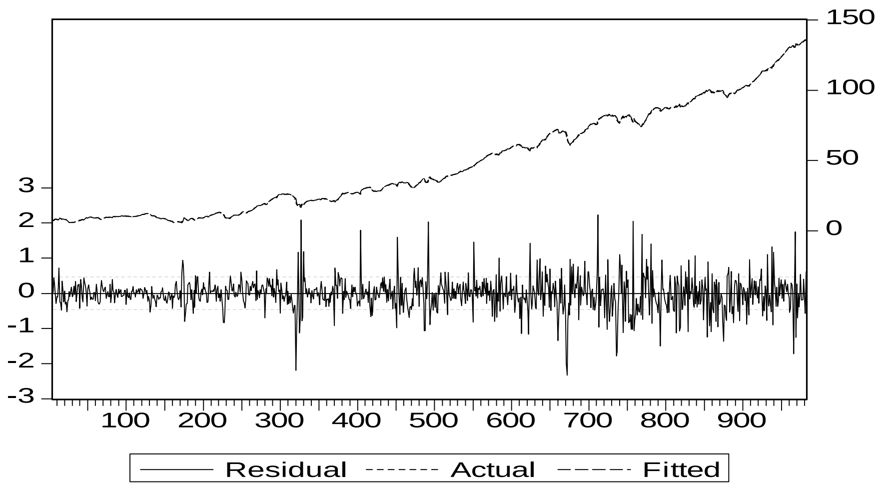

Fig. 1: industrial production: actual values, GARCH model, Residuals

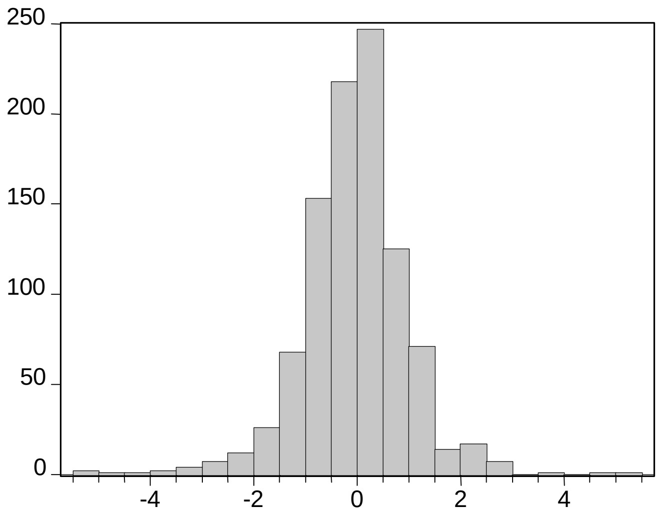

Fig. 2: Industrial production
Standardized Residuals

Observations 978	Sample 5 982
Mean	-0.048248
Median	-0.009958
Maximum	5.277011
Minimum	-5.217538
Std. Dev.	0.999358
Skewness	-0.239272
Kurtosis	6.706783

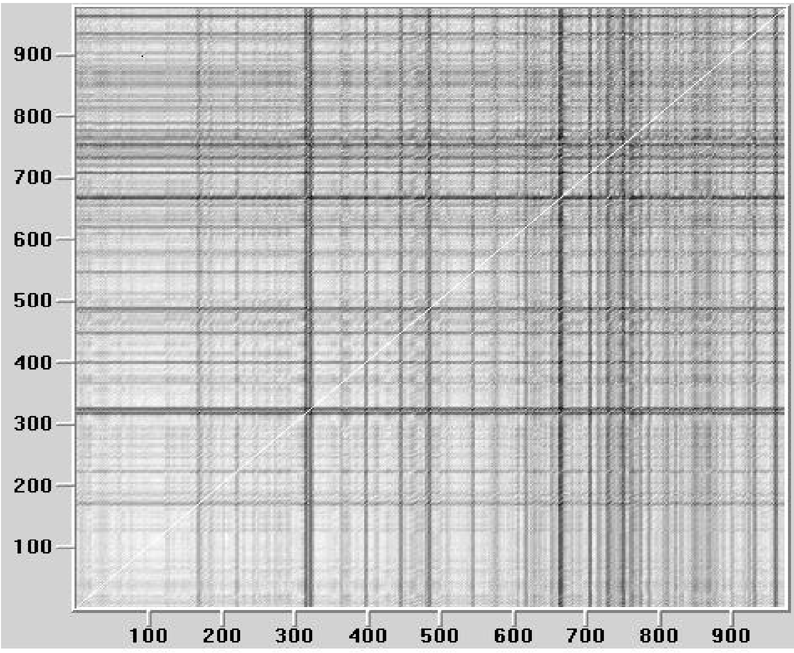

Fig. 3: industrial production, recurrence plot

Tab 2: industrial production							
Obs : N =981	SD/Spread=0.08						
ε	m	$C_{1,N}$ *N*(N-1)/2	$C_{m,N}$ *N*(N-1)/2	BDS	SD	$W_{m,N}$	d_m
0.2	2	411595	361646	0.51	0.06	8.10	0.17
0.2	4	411595	289045	1.81	0.18	10.24	0.31
0.2	6	411595	236374	2.84	0.26	10.82	0.44
0.2	8	411595	195377	3.47	0.31	11.05	0.56
0.2	10	411595	163324	3.83	0.34	11.38	0.67
0.12	2	327561	238317	0.93	0.10	9.55	0.33
0.12	4	327561	140982	2.37	0.18	13.53	0.58
0.12	6	327561	90856	2.75	0.17	16.42	0.79
0.12	8	327561	61569	2.55	0.13	19.74	0.98
0.12	10	327561	43908	2.20	0.09	24.65	1.14
0.08	2	232510	125399	0.82	0.08	9.80	0.52
0.08	4	232510	45353	1.23	0.08	16.00	0.91
0.08	6	232510	19305	0.86	0.04	22.73	1.25
0.08	8	232510	9328	0.52	0.01	34.62	1.53
0.08	10	232510	4898	0.30	0.01	56.58	1.78

Tab 3: indudrial production *after randomization*							
ε	m	$C_{1,N}$ *N*(N-1)/2	$C_{m,N}$ *N*(N-1)/2	BDS	SD	$W_{m,N}$	d_m
0.2	2	406397	348105	-0.08	0.06	-1.21	0.19
0.2	4	406397	255441	-0.17	0.18	-0.94	0.38
0.2	6	406397	187886	-0.18	0.26	-0.67	0.57
0.2	8	406397	137479	-0.22	0.31	-0.69	0.77
0.2	10	406397	101340	-0.17	0.34	-0.51	0.95
0.12	2	323688	219011	-0.17	0.10	-1.73	0.37
0.12	4	323688	99103	-0.31	0.18	-1.77	0.74
0.12	6	323688	44915	-0.25	0.17	-1.47	1.12
0.12	8	323688	19903	-0.19	0.13	-1.48	1.51
0.12	10	323688	9009	-0.11	0.09	-1.24	1.89
0.08	2	229970	109411	-0.16	0.08	-1.92	0.57
0.08	4	229970	24260	-0.15	0.08	-1.88	1.15
0.08	6	229970	5456	-0.05	0.04	-1.39	1.73
0.08	8	229970	1203	-0.02	0.02	-1.21	2.32
0.08	10	229970	283	0.00	0.01	-0.83	2.88

Tab 4: transportation equipment production			
ADF Test Statistic	-3.90	1% Critical Value	-3.97
		5% Critical Value	-3.42
		10% Critical Value	-3.13

Model: $Y(t) = 0.09+0.97*Y(t-1) + 6.46E\text{-}0.5*t+ \varepsilon_t$
$\sigma_t^2 = 4.11E\text{-}04+0.41\sigma_{t-1}^2 +0.31\varepsilon_{t-1}^2$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Y(t-1)	0.974117	0.008099	120.274	0
Intercept	0.0874	0.027695	3.15586	0.0016
TREND	6.46E-05	2.08E-05	3.106805	0.0019
C	0.000411	0.000142	2.904026	0.0037
ARCH(1)	0.407139	0.121736	3.344442	0.0008
GARCH(1)	0.31031	0.146487	2.118341	0.0341
R-squared	0.99463	Mean dependent var		4.088223
Adjusted R-squared	0.994587	S.D. dependent var		0.486078
S.E. of regression	0.035761	Akaike info criterion		-4.10689
Sum squared resid	0.801854	Schwarz criterion		-4.06471
Durbin-Watson stat	1.877671	5% Critical Value 3% Critical Value 1% Critical Value	2.13 2.16 2.19	1.86 1.84 1.81
ARCH Test on residuals:	Obs*R-squared 0.015523	Probability 0.900848	F-statistic 0.015474	Probability 0.901043

Tab 5: industrial machinery production			
ADF Test Statistic	-3.80	1% Critical Value	-3.98
		5% Critical Value	-3.42
		10% Critical Value	-3.13

Model:
$Y(t) = 0.04+0.98*Y(t-1) +0.31*[Y(t-1)-Y(t-2)]+0.30*[Y(t-2)-Y(t-3)]+ 7.26E\text{-}0.5*t+ \varepsilon_t$
$\sigma_t^2 = 4.49E\text{-}05+0.60\sigma_{t-1}^2 +0.15\varepsilon_{t-1}^2$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Y(t-1)	0.983132	0.003838	256.1834	0
Y(t-1)-Y(t-2)	0.313914	0.034976	8.975011	0
Y(t-2)-Y(t-3)	0.297845	0.036973	8.055791	0
Intercept	0.043703	0.0099	4.414574	0
TREND	7.26E-05	1.59E-05	4.576668	0
C	4.49E-05	1.97E-05	2.279031	0.0227
ARCH(1)	0.150593	0.052901	2.846704	0.0044
GARCH(1)	0.602048	0.136423	4.413111	0
R-squared	0.999669	Mean dependent var		3.854838
Adjusted R-squared	0.999665	S.D. dependent var		0.751639
S.E. of regression	0.013757	Akaike info criterion		-5.76243
Sum squared resid	0.117709	Schwarz criterion		-5.70597
Durbin-Watson stat	1.872385	5% Critical Value 3% Critical Value 1% Critical Value	2.13 2.16 2.19	1.86 1.84 1.81
ARCH Test on residuals:	Obs*R-squared 2.503256	Probability 0.286039	F-statistic 1.250634	Probability 0.287039

Tab 6: electric machinery production			
ADF Test Statistic	-2.77	1% Critical Value	-3.98
		5% Critical Value	-3.42
		10% Critical Value	-3.13

Model:
$Y(t) = 0.07+0.96*Y(t-1) +0.16*[Y(t-1)-Y(t-2)]+0.21*[Y(t-2)-Y(t-3)]+ 2.15E\text{-}0.4*t+ \varepsilon_t$
$\sigma_t^2 = 8.36E\text{-}05+0.50\sigma_{t-1}^2 +0.29\varepsilon_{t-1}^2$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Y(t-1)	0.96001	0.007401	129.7157	0
Y(t-1)-Y(t-2)	0.160769	0.046921	3.426395	0.0006
Y(t-2)-Y(t-3)	0.207996	0.042557	4.887463	0
Intercept	0.07286	0.012766	5.707498	0
TREND	0.000215	4.07E-05	5.277838	0
C	8.36E-05	2.24E-05	3.733987	0.0002
ARCH(1)	0.288022	0.071686	4.017817	0.0001
GARCH(1)	0.509315	0.092626	5.498605	0
R-squared	0.999269	Mean dependent var		2.989646
Adjusted R-squared	0.999258	S.D. dependent var		0.735146
S.E. of regression	0.020027	Akaike info criterion		-5.125052
Sum squared resid	0.182896	Schwarz criterion		-5.053674
Durbin-Watson stat	1.810907	5% Critical Value 3% Critical Value 1% Critical Value	2.15 2.18 2.21	1.85 1.81 1.78
ARCH Test on residuals:	Obs*R-squared 0.205277	Probability 0.650495	F-statistic 0.204481	Probability 0.651341

Tab 7: Hi-Tech					
ADF Test Statistic	0.578766	1% Critical Value			-3.9854
		5% Critical Value			-3.4230
		10% Critical Value			-3.1341

Model:
$Y(t) = 1.00*Y(t-1) +0.27*[Y(t-1)-Y(t-2)]+0.29*[Y(t-2)-Y(t-3)]+ 3.89E-0.5*t+ \varepsilon_t$
$\sigma_t^2 = 3.89E-05+0.68\sigma_{t-1}^2 +0.12\varepsilon_{t-1}^2$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Y(t-1)	1.001579	0.00027	3715.943	0
Y(t-1)-Y(t-2)	0.265929	0.056883	4.675001	0
Y(t-2)-Y(t-3)	0.292968	0.056174	5.215386	0
TREND	3.89E-05	2.04E-05	1.908155	0.0564
ARCH(1)	0.122328	0.048225	2.53659	0.0112
GARCH(1)	0.677888	0.13518	5.014715	0
R-squared	0.999896	Mean dependent var		3.626309
Adjusted R-squared	0.999895	S.D. dependent var		1.360402
S.E. of regression	0.013931	Akaike info criterion		-5.749052
Sum squared resid	0.074525	Schwarz criterion		-5.688035
Durbin-Watson stat	2.031761	5% Critical Value	2.17	1.83
		3% Critical Value	2.20	1.80
		1% Critical Value	2.23	1.74
ARCH Test on residuals:	Obs*R-squared 0.14387	Probability 0.704464	F-statistic 0.143183	Probability 0.705345

tab 8: employment					
ADF Test Statistic	-4.205271	1% Critical Value			-3.9754
		5% Critical Value			-3.4182
		10% Critical Value			-3.1313

Model:
$Y(t) = 0.19+0.98*Y(t-1) +0.23*[Y(t-1)-Y(t-2)]+0.23*[Y(t-2)-Y(t-3)]+0.17*[Y(t-3)-Y(t-4)] +3.11E-0.5*t+ \varepsilon_t$
$\sigma_t^2 = 9.41E-06+0.60\sigma_{t-1}^2 +0.15\varepsilon_{t-1}^2$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Y(t-1)	0.982307	0.003917	250.7972	0
Y(t-1)-Y(t-2)	0.229051	0.061435	3.728361	0.0002
Y(t-2)-Y(t-3)	0.223686	0.046996	4.75968	0
Y(t-3)-Y(t-4)	0.165958	0.055108	3.011501	0.0026
Intercept	0.186283	0.040823	4.563192	0
TREND	3.11E-05	7.39E-06	4.214051	0
C	9.41E-06	3.23E-06	2.916529	0.0035
ARCH(1)	0.15	0.111295	1.347765	0.1777
GARCH(1)	0.6	0.117113	5.123254	0
R-squared	0.999899	Mean dependent var		11.12976
Adjusted R-squared	0.999898	S.D. dependent var		0.379236
S.E. of regression	0.003829	Akaike info criterion		-8.245976
Sum squared resid	0.010481	Schwarz criterion		-8.188983
Durbin-Watson stat	2.020042	5% Critical Value	2.12	1.88
		3% Critical Value	2.15	1.85
		1% Critical Value	2.17	1.83
ARCH Test on residuals:	Obs*R-squared 0.563179	Probability 0.452982	F-statistic 0.562058	Probability 0.453677

tab 9: hourly earnings of production workers					
ADF Test Statistic	-1.066504	1% Critical Value			-3.9742
		5% Critical Value			-3.4176
		10% Critical Value			-3.1309

Model:
$Y(t) = 0.01+1.00*Y(t-1) +0.13*[Y(t-1)-Y(t-2)]+0.09*[Y(t-2)-Y(t-3)]+0.18*[Y(t-3)-Y(t-4)] - 4.33E-0.5*t+ \varepsilon_t$
$\sigma_t^2 = 5.67E-06+0.71\sigma_{t-1}^2 +0.21\varepsilon_{t-1}^2$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Y(t-1)	1.009338	0.001686	598.5128	0
Y(t-1)-Y(t-2)	0.126885	0.061628	2.058879	0.0395
Y(t-2)-Y(t-3)	0.091454	0.048557	1.883432	0.0596
Y(t-3)-Y(t-4)	0.189606	0.04888	3.879028	0.0001
Intercept	0.01023	0.001032	9.915531	0
TREND	-4.33E-05	6.93E-06	-6.254606	0
C	5.67E-06	1.69E-06	3.362442	0.0008
ARCH(1)	0.212312	0.148577	1.428968	0.153
GARCH(1)	0.709285	0.080719	8.787132	0
R-squared	0.999929	Mean dependent var		1.075365
Adjusted R-squared	0.999929	S.D. dependent var		1.038719
S.E. of regression	0.008779	Akaike info criterion		-7.457538
Sum squared resid	0.061578	Schwarz criterion		-7.405247
Durbin-Watson stat	1.985708	5% Critical Value	2.12	1.88
		3% Critical Value	2.14	1.86
		1% Critical Value	2.16	1.84
ARCH Test on residuals:	Obs*R-squared 0.628651	Probability 0.427851	F-statistic 0.627582	Probability 0.428477

Tab 10: Consumer Price Index				
ADF Test Statistic	-0.846908	1% Critical Value		-3.9719
		5% Critical Value		-3.4165
		10% Critical Value		-3.1302
Model: Y(t) = -5.17E-03+1.00*Y(t-1) +0.31*[Y(t-1)-Y(t-2)]+0.15*[Y(t-2)-Y(t-3)]+0.22*[Y(t-3)-Y(t-4)] -7.57E-0.6*t+ ε_t σ_t^2 =6.20E-06+0.60σ_{t-1}^2 +0.15ε_{t-1}^2				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
Y(t-1)	1.002717	0.000524	1915.222	0
Y(t-1)-Y(t-2)	0.314714	0.04027	7.815017	0
Y(t-2)-Y(t-3)	0.154939	0.042976	3.605285	0.0003
Y(t-3)-Y(t-4)	0.219484	0.051788	4.238104	0
Intercept	-0.005174	0.000989	-5.229768	0
TREND	-7.57E-06	1.80E-06	-4.208366	0
C	6.20E-06	2.99E-06	2.075324	0.038
ARCH(1)	0.153316	0.123373	1.242702	0.214
GARCH(1)	0.602011	0.192731	3.123577	0.0018
R-squared	0.99995	Mean dependent var		3.510637
Adjusted R-squared	0.99995	S.D. dependent var		0.836057
S.E. of regression	0.005927	Akaike info criterion		-7.71261
Sum squared resid	0.036119	Schwarz criterion		-7.6697
Durbin-Watson stat	1.969541	5% Critical Value	2.10	1.90
		3% Critical Value	2.12	1.88
		1% Critical Value	2.14	1.86
ARCH Test on residuals:	Obs*R-squared 2.473278	Probability 0.115796	F-statistic 2.474411	Probability 0.116019

Tab 11: maximal liapunov exponents	M=1	M=2	M=3	M=4	M=5
Uniform i.i.d. process	3.40	1.41	1.24	0.77	0.85
Gaussian i.i.d. process	2.53	0.75	0.44	0.33	0.30
Industrial production	2.61	0.81	0.45	0.32	0.31
transportation eq. production	2.00	0.66	0.44	0.35	0.35
industrial machinery and eq.	1.93	0.63	0.35	0.28	0.26
electrical machinery	1.70	0.57	0.32	0.23	0.27
Hi-Tech	1.58	0.52	0.30	0.28	0.27
employment	1.55	0.67	036	0.30	0.27
hourly earnings	1.92	1.45	0.94	0.80	0.66
consumer price index	2.13	0.80	0.46	0.34	0.30

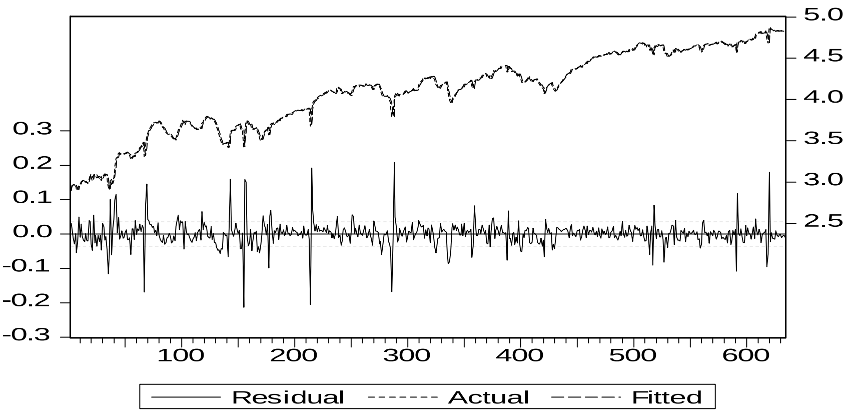

Fig. 4: Transportation equipment production: actual values, GARCH model, Residuals

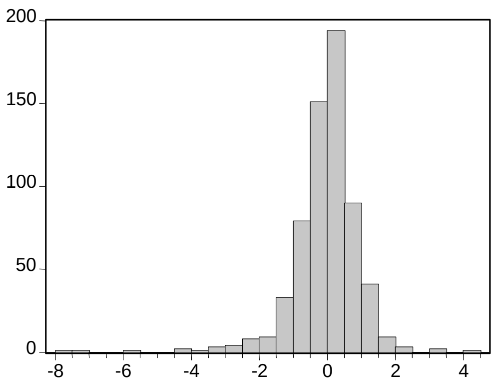

Fig 5: Transpotation eq. prod.
Series: Standardized Residuals
Observations 633 Sample 2 634

Mean	-0.027365
Median	0.052255
Maximum	4.447432
Minimum	-7.637314
Std. Dev.	0.989127
Skewness	-1.952752
Kurtosis	15.93459

i

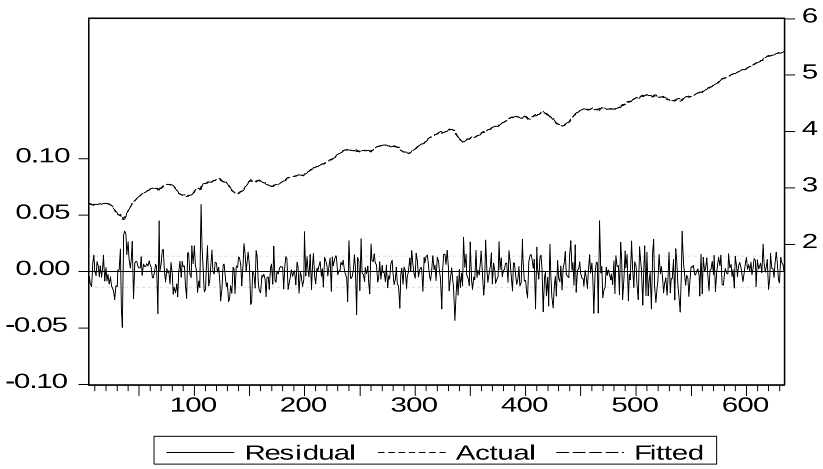

Fig. 6: Industrial machinery production: actual values, GARCH model, Residuals

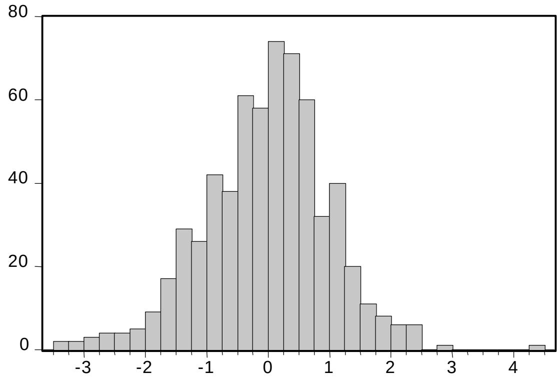

Fig 7: industrial mach. prod.
Standardized Residuals

Observations 630 Sample 5 634

Mean	-0.029108
Median	0.052560
Maximum	4.468858
Minimum	-3.397197
Std. Dev.	1.015118
Skewness	-0.195088
Kurtosis	3.844262

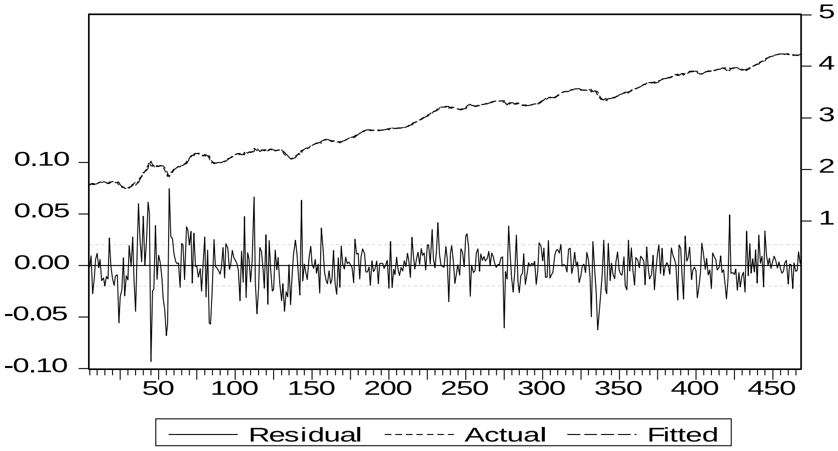

Fig. 8: Electric machinery production: actual values, GARCH model, Residuals

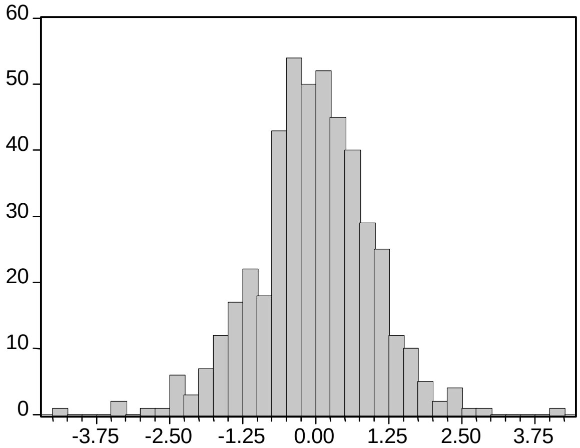

Fig. 9: electric machinery production:
Standardized Residuals

Observations 464 Sample 5 468

Mean	-0.036564
Median	-0.021975
Maximum	4.045548
Minimum	-4.285570
Std. Dev.	1.000011
Skewness	-0.184560
Kurtosis	4.426845

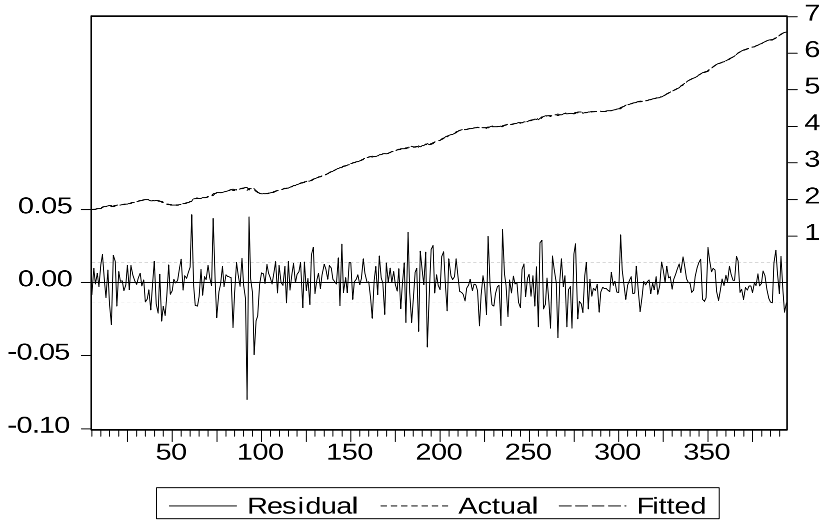

Fig. 10: Hi-Tech production: actual values, GARCH model, Residuals

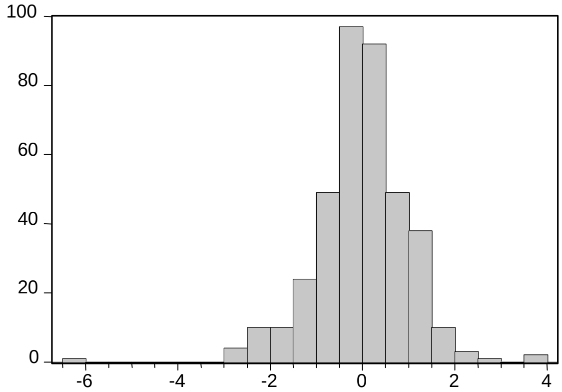

Fig. 11 Hi-Tech production:
Standardized Residuals

Observations 390	Sample 5 394
Mean	-0.006667
Median	-0.002427
Maximum	3.837274
Minimum	-6.226926
Std. Dev.	1.001157
Skewness	-0.570892
Kurtosis	7.285860

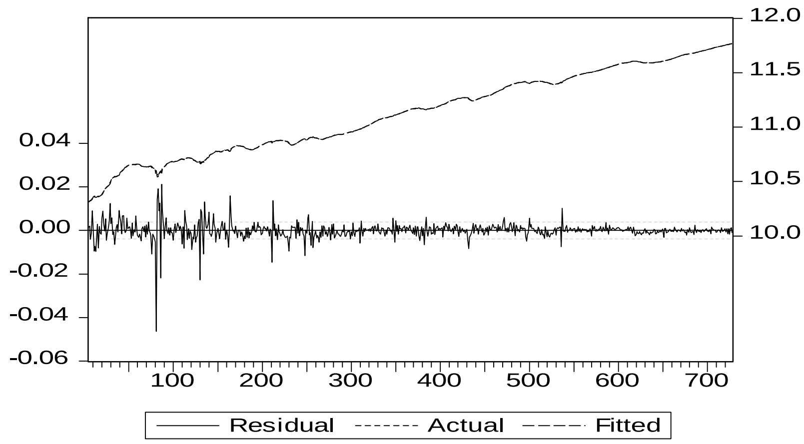

Fig. 12: Employment: actual level, GARCH model, Residuals

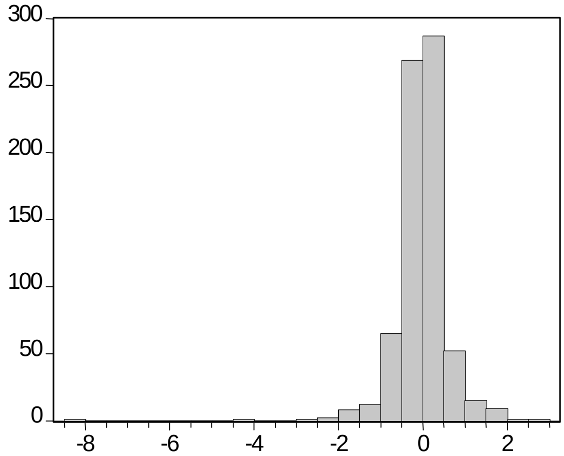

Fig. 13: Employment
Standardized Residuals

Observations 724	Sample 5 728
Mean	-0.015987
Median	0.002652
Maximum	2.907456
Minimum	-8.182487
Std. Dev.	0.638225
Skewness	-3.252118
Kurtosis	43.45859

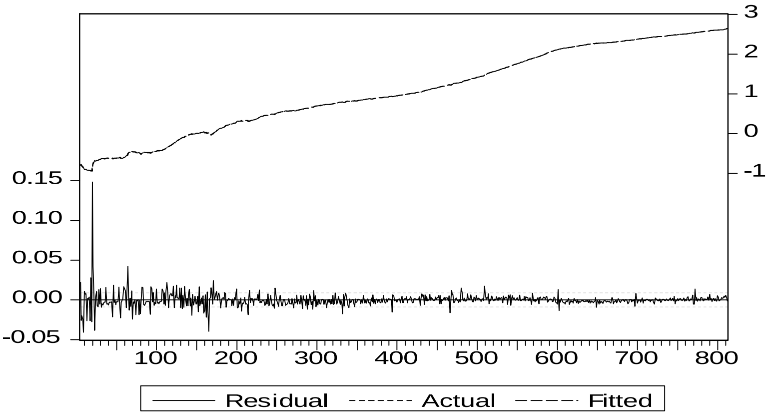

Fig. 14: Hourly wages: actual level, GARCH model, Residuals

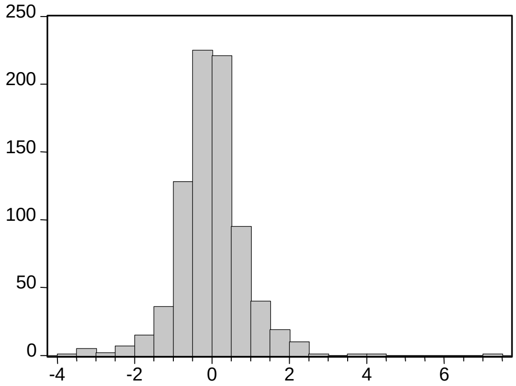

Fig. 15: Hourly wages
Standardized Residuals

Observations 808	Sample 5 812
Mean	-0.023235
Median	-0.032950
Maximum	7.321643
Minimum	-3.595514
Std. Dev.	0.877188
Skewness	0.710590
Kurtosis	11.10385

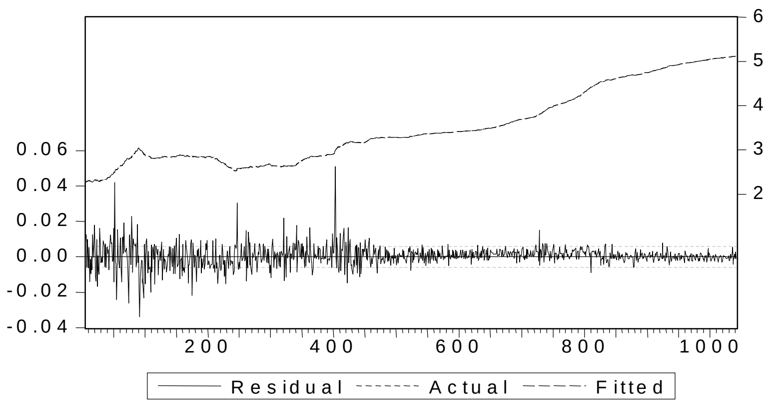

Fig. 16: Consumer price index: actual level, GARCH model, Residuals

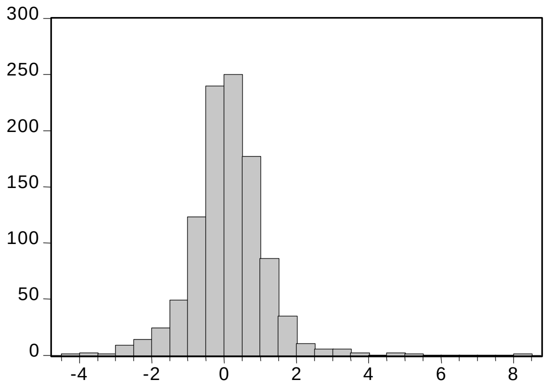

Fig. 17: Consumer price index
Standardized Residuals

Observations	Sample 6 1042
Mean	0.114499
Median	0.085245
Maximum	8.297400
Minimum	-4.077784
Std. Dev.	1.013420
Skewness	0.615866
Kurtosis	9.280853
Jarque-Bera	1770.084
Probability	0.000000

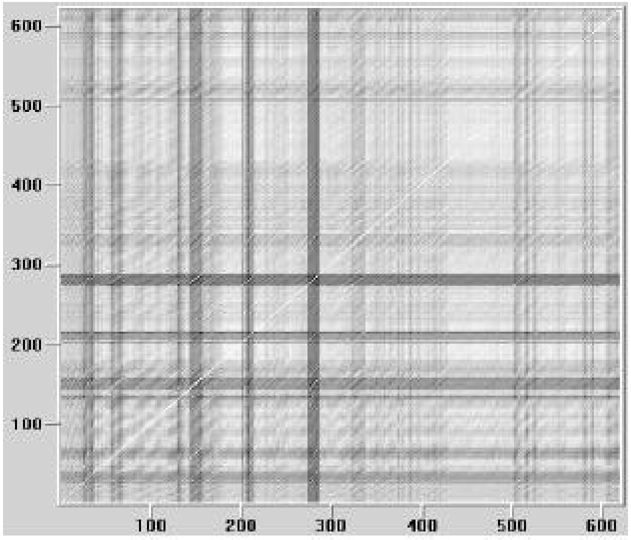

Fig. 18: Transportation equipment production, recurrence plot

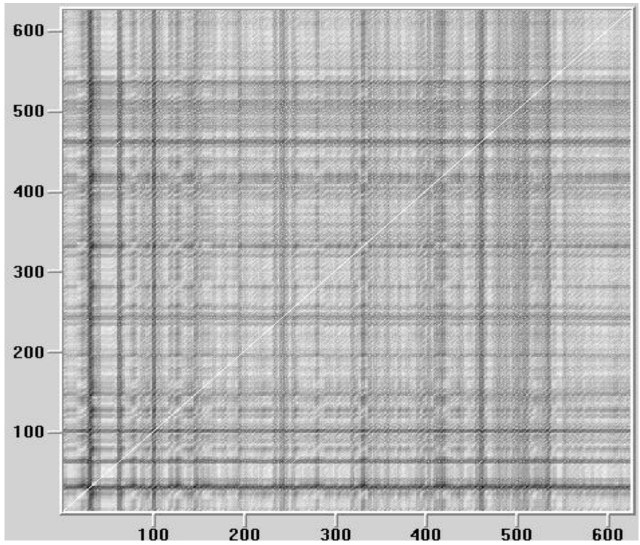

Fig. 19: industrial machinery production, recurrence plot

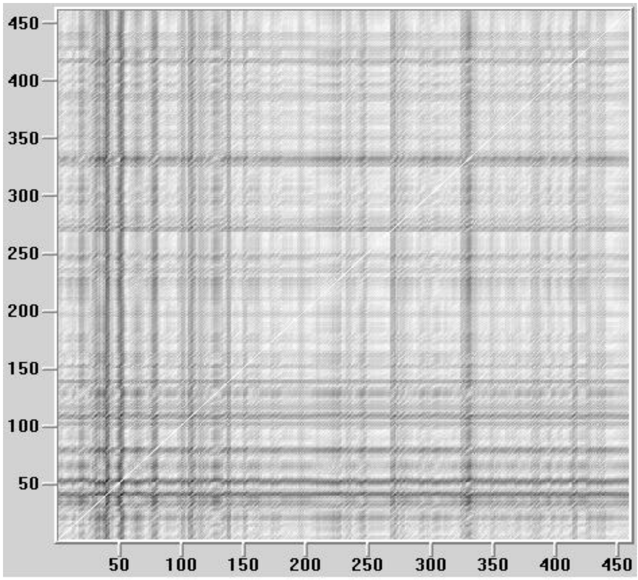

Fig. 20: electric machinery production, recurrence plot

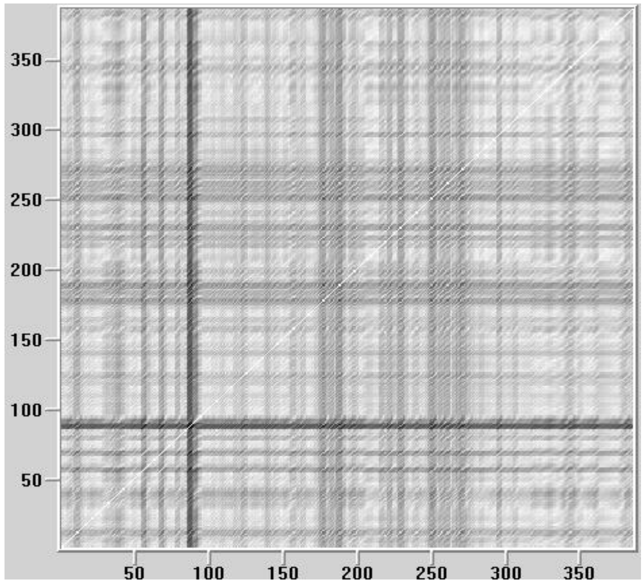

Fig. 21: Hi-Tech production, recurrence plot

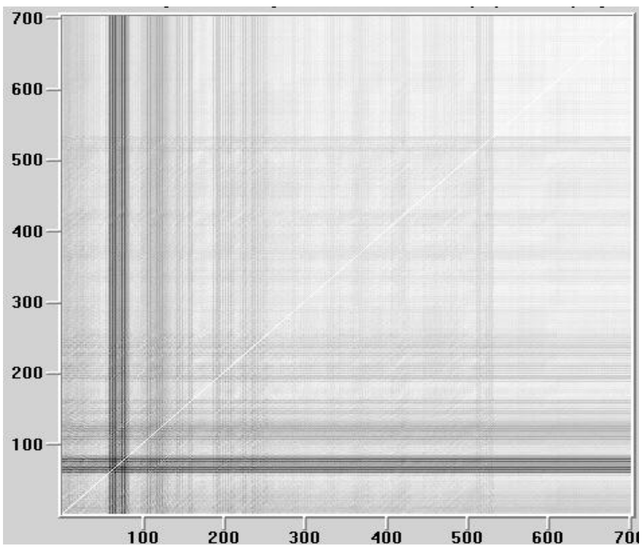

Fig. 22: employment, recurrence plot

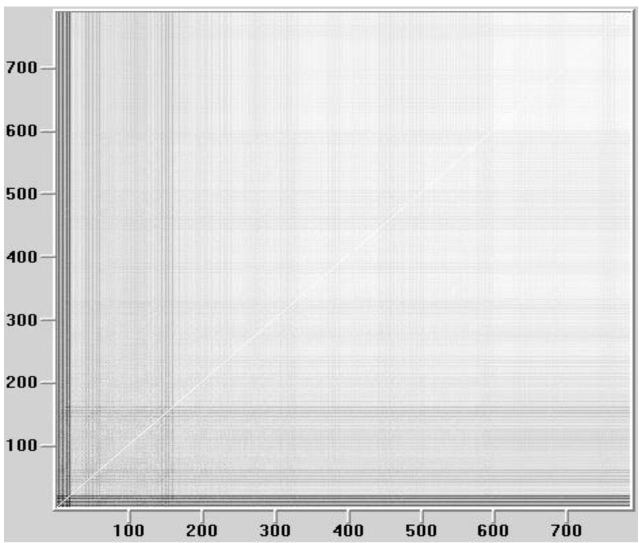

Fig. 23: hourly earnings, recurrence plot

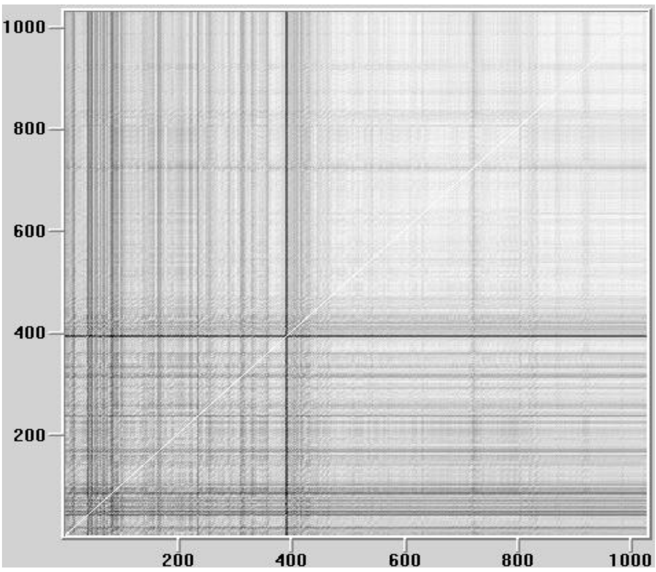

Fig. 24: consumer price index, recurrence plot

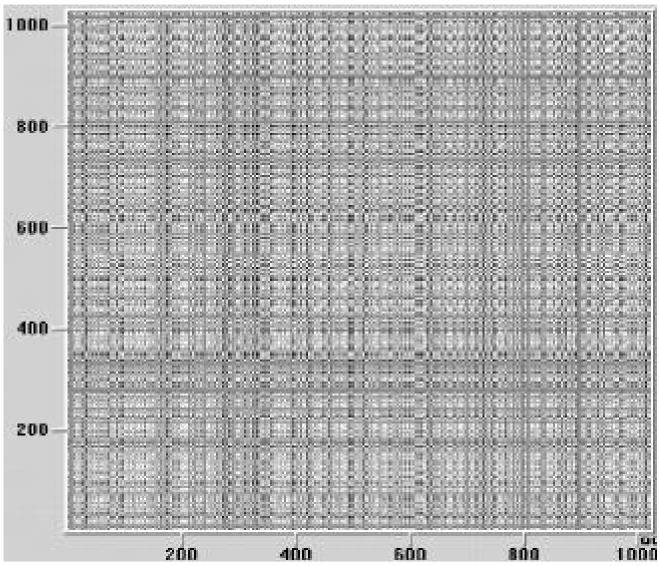

Fig. 25: recurrence plot of pseudo rasndom numbers from a uniform *i.i.d.* distribution

Tab 12:		transportation eq. production					
Obs : N=633		SD/Spread=0.08					
ε	m	$C_{1,N}$ *N*(N-1)/2	$C_{m,N}$ *N*(N-1)/2	BDS	SD	$W_{m,N}$	d_m
0.16	2	177341	161933	0.66	0.07	9.92	0.12
0.16	4	177341	138401	2.03	0.20	10.31	0.20
0.16	6	177341	118015	2.85	0.31	9.22	0.29
0.16	8	177341	99941	3.23	0.39	8.24	0.38
0.16	10	177341	84470	3.36	0.44	7.56	0.47
0.09	2	144660	113856	1.21	0.10	11.55	0.24
0.09	4	144660	74608	2.59	0.21	12.37	0.41
0.09	6	144660	49263	2.69	0.22	12.08	0.59
0.09	8	144660	32803	2.32	0.19	12.19	0.76
0.09	10	144660	21981	1.85	0.15	12.63	0.93
0.05	2	101004	59348	1.07	0.09	11.46	0.41
0.05	4	101004	22808	1.26	0.09	13.51	0.74
0.05	6	101004	8921	0.72	0.05	14.58	1.06
0.05	8	101004	3873	0.39	0.02	18.41	1.35
0.05	10	101004	1766	0.20	0.01	24.60	1.61

Tab 13:		industrial machinery production					
Obs : N=633		SD/Spread=0.15					
ε	m	$C_{1,N}$ *N*(N-1)/2	$C_{m,N}$ *N*(N-1)/2	BDS	SD	$W_{m,N}$	d_m
0.2	2	149659	114908	0.28	0.07	3.98	0.34
0.2	4	149659	68946	0.65	0.15	4.20	0.66
0.2	6	149659	42685	0.83	0.18	4.70	0.95
0.2	8	149659	27111	0.85	0.16	5.18	1.24
0.2	10	149659	17504	0.76	0.14	5.61	1.51
0.12	2	106384	59251	0.29	0.07	4.06	0.58
0.12	4	106384	18993	0.34	0.08	4.26	1.12
0.12	6	106384	6470	0.23	0.05	4.86	1.64
0.12	8	106384	2268	0.12	0.02	5.30	2.14
0.12	10	106384	801	0.05	0.01	5.61	2.63
0.08	2	69972	25954	0.17	0.04	3.87	0.79
0.08	4	69972	3736	0.09	0.02	4.13	1.54
0.08	6	69972	583	0.03	0.01	4.79	2.26
0.08	8	69972	86	0.01	0.00	4.39	3.01
0.08	10	69972	15	0.00	0.00	5.52	3.68

Tab 14:		electrical machinery production					
Obs : N=633		SD/Spread=0.12					
ε	m	$C_{1,N}$ *N*(N-1)/2	$C_{m,N}$ *N*(N-1)/2	BDS	SD	$W_{m,N}$	d_m
0.2	2	85710	70416	0.47	0.07	6.34	0.26
0.2	4	85710	49753	1.37	0.18	7.72	0.48
0.2	6	85710	36053	1.82	0.23	7.99	0.68
0.2	8	85710	26631	1.95	0.23	8.31	0.87
0.2	10	85710	19800	1.86	0.22	8.57	1.05
0.12	2	63306	40057	0.59	0.08	6.89	0.47
0.12	4	63306	17844	1.02	0.11	9.01	0.86
0.12	6	63306	8275	0.79	0.08	9.76	1.23
0.12	8	63306	3921	0.49	0.05	10.64	1.58
0.12	10	63306	1848	0.27	0.02	11.49	1.94
0.08	2	42416	18661	0.40	0.06	7.17	0.68
0.08	4	42416	4323	0.35	0.03	10.41	1.25
0.08	6	42416	1058	0.13	0.01	12.18	1.79
0.08	8	42416	281	0.05	0.00	15.54	2.31
0.08	10	42416	92	0.02	0.00	24.97	2.74

Tab 15:		HI-TECH					
Obs : N=393		SD/Spread=0.16					
ε	m	$C_{1,N}$ *N*(N-1)/2	$C_{m,N}$ *N*(N-1)/2	BDS	SD	$W_{m,N}$	d_m
0.2	2	62853	52644	0.22	0.06	3.35	0.23
0.2	4	62853	37901	0.71	0.17	4.21	0.43
0.2	6	62853	28258	1.15	0.23	4.95	0.61
0.2	8	62853	21214	1.33	0.26	5.15	0.79
0.2	10	62853	16094	1.36	0.26	5.32	0.96
0.12	2	47571	31242	0.41	0.09	4.63	0.42
0.12	4	47571	14411	0.75	0.13	5.65	0.79
0.12	6	47571	7053	0.68	0.11	6.34	1.14
0.12	8	47571	3473	0.46	0.07	6.65	1.47
0.12	10	47571	1685	0.27	0.04	6.73	1.82
0.08	2	32490	15031	0.31	0.07	4.75	0.63
0.08	4	32490	3655	0.30	0.05	6.42	1.18
0.08	6	32490	944	0.13	0.02	7.22	1.70
0.08	8	32490	279	0.05	0.01	9.48	2.18
0.08	10	32490	76	0.02	0.00	10.72	2.68

Tab 16		Employment					
Obs : N=729		SD/Spread=0.07					
ε	m	$C_{1,N}$ *N*(N-1)/2	$C_{m,N}$ *N*(N-1)/2	BDS	SD	$W_{m,N}$	d_m
0.1	2	230569	210595	0.83	0.07	11.61	0.09
0.1	4	230569	179862	2.43	0.21	11.64	0.16
0.1	6	230569	158349	3.92	0.33	12.03	0.22
0.1	8	230569	141379	5.03	0.41	12.34	0.27
0.1	10	230569	128152	5.90	0.46	12.90	0.31
0.05	2	179060	132962	1.11	0.11	9.84	0.23
0.05	4	179060	82858	2.69	0.20	13.22	0.38
0.05	6	179060	59024	3.38	0.19	17.37	0.50
0.05	8	179060	44535	3.36	0.15	22.37	0.59
0.05	10	179060	34742	3.04	0.10	29.28	0.67
0.03	2	109230	53095	0.79	0.08	10.16	0.43
0.03	4	109230	17147	0.96	0.05	18.07	0.74
0.03	6	109230	7424	0.63	0.02	32.38	0.97
0.03	8	109230	3902	0.38	0.01	66.41	1.14
0.03	10	109230	2170	0.22	0.00	145.63	1.30

Tab 17		Hourly earnings of production workers					
Obs : N=811		SD/Spread=0.05					
ε	m	$C_{1,N}$ *N*(N-1)/2	$C_{m,N}$ *N*(N-1)/2	BDS	SD	$W_{m,N}$	d_m
0.1	2	303634	287595	0.48	0.04	11.81	0.05
0.1	4	303634	261631	1.59	0.13	11.98	0.10
0.1	6	303634	242938	2.89	0.23	12.55	0.13
0.1	8	303634	228276	4.15	0.32	12.94	0.15
0.1	10	303634	216534	5.32	0.40	13.35	0.18
0.05	2	250446	204795	1.13	0.11	10.68	0.16
0.05	4	250446	148850	3.17	0.24	13.36	0.26
0.05	6	250446	116198	4.40	0.28	15.51	0.34
0.05	8	250446	94096	4.88	0.27	17.83	0.42
0.05	10	250446	79655	5.04	0.24	21.34	0.47
0.03	2	167734	101009	1.30	0.10	12.52	0.32
0.03	4	167734	45591	2.01	0.11	18.65	0.53
0.03	6	167734	24154	1.60	0.06	26.89	0.71
0.03	8	167734	13560	1.06	0.03	39.89	0.86
0.03	10	167734	8101	0.68	0.01	64.14	1.00

Tab 18	c.p.i						
Obs : N=1041		SD/Spread=0.08					
ε	m	$C_{1,N}$ *N*(N-1)/2	$C_{m,N}$ *N*(N-1)/2	BDS	SD	$W_{m,N}$	d_m
0.13	2	462040	406138	0.57	0.07	8.61	0.14
0.13	4	462040	328457	2.18	0.18	11.76	0.24
0.13	6	462040	279585	3.87	0.27	14.12	0.32
0.13	8	462040	248799	5.45	0.33	16.65	0.38
0.13	10	462040	225992	6.62	0.35	18.92	0.42
0.07	2	342027	238672	1.28	0.10	12.49	0.31
0.07	4	342027	140553	3.18	0.16	19.88	0.51
0.07	6	342027	97769	3.76	0.13	28.38	0.64
0.07	8	342027	75419	3.70	0.09	41.70	0.74
0.07	10	342027	60409	3.32	0.05	62.39	0.82
0.04	2	215075	103623	1.06	0.07	15.04	0.50
0.04	4	215075	32849	1.15	0.04	25.89	0.86
0.04	6	215075	13134	0.66	0.01	44.14	1.14
0.04	8	215075	6059	0.35	0.00	84.85	1.37
0.04	10	215075	3088	0.18	0.00	184.93	1.58

Tab 19		Shuffled transportation eq. production					
ε	m	$C_{1,N}$ *N*(N-1)/2	$C_{m,N}$ *N*(N-1)/2	BDS	SD	$W_{m,N}$	d_m
0.16	2	172558	152063	-0.02	0.07	-0.28	0.14
0.16	4	172558	118389	-0.01	0.20	-0.03	0.27
0.16	6	172558	92085	-0.01	0.31	-0.02	0.41
0.16	8	172558	72678	0.13	0.39	0.33	0.54
0.16	10	172558	57437	0.22	0.44	0.49	0.67
0.09	2	140365	100687	0.00	0.10	-0.03	0.28
0.09	4	140365	52907	0.14	0.21	0.66	0.55
0.09	6	140365	27588	0.12	0.22	0.53	0.82
0.09	8	140365	14840	0.14	0.19	0.77	1.08
0.09	10	140365	7887	0.11	0.14	0.76	1.35
0.05	2	97643	48448	-0.04	0.09	-0.40	0.47
0.05	4	97643	12638	0.06	0.09	0.71	0.93
0.05	6	97643	3319	0.04	0.05	0.81	1.39
0.05	8	97643	891	0.02	0.02	0.90	1.84
0.05	10	97643	240	0.01	0.01	0.91	2.29

Tab 20		Shuffled machinery eq. production					
ε	m	$C_{1,N}$ *N*(N-1)/2	$C_{m,N}$ *N*(N-1)/2	BDS	SD	$W_{m,N}$	d_m
0.20	2	147062	110417	-0.02	0.07	-0.25	0.35
0.20	4	147062	61827	-0.08	0.15	-0.55	0.71
0.20	6	147062	34463	-0.11	0.18	-0.62	1.08
0.20	8	147062	18540	-0.18	0.16	-1.13	1.46
0.20	10	147062	9727	-0.20	0.13	-1.51	1.86
0.12	2	104451	55557	-0.03	0.07	-0.38	0.60
0.12	4	104451	15518	-0.05	0.08	-0.62	1.21
0.12	6	104451	4340	-0.02	0.05	-0.53	1.82
0.12	8	104451	1139	-0.02	0.02	-0.90	2.46
0.12	10	104451	287	-0.01	0.01	-1.13	3.12
0.08	2	68682	23851	-0.03	0.04	-0.78	0.82
0.08	4	68682	2740	-0.03	0.02	-1.43	1.66
0.08	6	68682	337	0.00	0.01	-0.70	2.47
0.08	8	68682	41	0.00	0.00	-0.48	3.29
0.08	10	68682	1	0.00	0.00	-2.84	4.73

Tab 21		Shuffled electrical machinery					
ε	m	$C_{1,N}$ *N*(N-1)/2	$C_{m,N}$ *N*(N-1)/2	BDS	SD	$W_{m,N}$	d_m
0.20	2	80029	62769	-0.07	0.07	-0.98	0.30
0.20	4	80029	38655	-0.13	0.18	-0.72	0.60
0.20	6	80029	23561	-0.18	0.22	-0.81	0.90
0.20	8	80029	14419	-0.16	0.23	-0.72	1.21
0.20	10	80029	8529	-0.20	0.21	-0.96	1.54
0.12	2	58806	33454	-0.13	0.08	-1.58	0.53
0.12	4	58806	10892	-0.12	0.11	-1.08	1.06
0.12	6	58806	3453	-0.08	0.08	-1.10	1.61
0.12	8	58806	1131	-0.03	0.04	-0.81	2.15
0.12	10	58806	359	-0.02	0.02	-0.76	2.70
0.08	2	39255	14780	-0.09	0.05	-1.60	0.75
0.08	4	39255	2061	-0.04	0.03	-1.42	1.51
0.08	6	39255	281	-0.01	0.01	-1.27	2.29
0.08	8	39255	42	0.00	0.00	-0.76	3.02
0.08	10	39255	6	0.00	0.00	-0.61	3.78

Tab 22		Shuffled Hi-Tech					
ε	m	$C_{1,N}$ *N*(N-1)/2	$C_{m,N}$ *N*(N-1)/2	BDS	SD	$W_{m,N}$	d_m
0.20	2	60924	50005	-0.06	0.07	-0.84	0.24
0.20	4	60924	33459	-0.17	0.17	-1.02	0.49
0.20	6	60924	21733	-0.39	0.24	-1.66	0.76
0.20	8	60924	14428	-0.36	0.26	-1.37	1.01
0.20	10	60924	9830	-0.24	0.26	-0.91	1.25
0.12	2	46240	28872	-0.01	0.09	-0.16	0.45
0.12	4	46240	11197	-0.03	0.14	-0.24	0.90
0.12	6	46240	3995	-0.12	0.11	-1.07	1.39
0.12	8	46240	1499	-0.06	0.07	-0.90	1.86
0.12	10	46240	590	-0.02	0.04	-0.58	2.31
0.08	2	31631	13566	0.01	0.07	0.12	0.66
0.08	4	31631	2483	0.00	0.05	0.03	1.32
0.08	6	31631	392	-0.02	0.02	-0.90	2.03
0.08	8	31631	53	-0.01	0.01	-1.42	2.81
0.08	10	31631	9	0.00	0.00	-1.06	3.50

Tab 23		Shuffled employment					
ε	m	$C_{1,N}$ *N*(N-1)/2	$C_{m,N}$ *N*(N-1)/2	BDS	SD	$W_{m,N}$	d_m
0.10	2	227858	199499	-5.49E-02	7.16E-02	-7.67E-01	0.11
0.10	4	227858	152004	-2.24E-01	2.10E-01	-1.07E+00	0.23
0.10	6	227858	115652	-3.28E-01	3.26E-01	-1.01E+00	0.35
0.10	8	227858	89030	-2.64E-01	4.08E-01	-6.47E-01	0.46
0.10	10	227858	68608	-2.04E-01	4.56E-01	-4.47E-01	0.58
0.05	2	176738	119070	-1.32E-01	1.13E-01	-1.17E+00	0.26
0.05	4	176738	53835	-2.04E-01	2.03E-01	-1.01E+00	0.52
0.05	6	176738	23350	-2.64E-01	1.93E-01	-1.36E+00	0.80
0.05	8	176738	10639	-1.42E-01	1.49E-01	-9.58E-01	1.07
0.05	10	176738	4963	-6.31E-02	1.02E-01	-6.17E-01	1.32
0.03	2	107740	43840	-9.16E-02	7.73E-02	-1.18E+00	0.48
0.03	4	107740	7127	-6.03E-02	5.28E-02	-1.14E+00	0.98
0.03	6	107740	987	-3.57E-02	1.92E-02	-1.86E+00	1.51
0.03	8	107740	149	-8.39E-03	5.67E-03	-1.48E+00	2.02
0.03	10	107740	26	-1.42E-03	1.50E-03	-9.44E-01	2.50

Tab 24		Shuffled hourly earnings of prduction workers					
ε	m	$C_{1,N}$ *N*(N-1)/2	$C_{m,N}$ *N*(N-1)/2	BDS	SD	$W_{m,N}$	d_m
0.10	2	297431	276887	0.07	0.04	1.68	0.06
0.10	4	297431	239692	0.16	0.13	1.17	0.12
0.10	6	297431	206709	0.15	0.23	0.65	0.19
0.10	8	297431	178461	0.16	0.32	0.50	0.25
0.10	10	297431	154444	0.20	0.40	0.49	0.32
0.05	2	244999	190013	0.24	0.11	2.23	0.17
0.05	4	244999	115436	0.52	0.24	2.21	0.34
0.05	6	244999	68797	0.42	0.28	1.50	0.51
0.05	8	244999	40386	0.26	0.27	0.97	0.69
0.05	10	244999	23804	0.17	0.23	0.74	0.87
0.03	2	164183	86015	0.17	0.10	1.59	0.36
0.03	4	164183	24878	0.25	0.11	2.29	0.69
0.03	6	164183	7295	0.13	0.06	2.24	1.03
0.03	8	164183	2002	0.04	0.03	1.62	1.38
0.03	10	164183	506	0.01	0.01	0.90	1.75

Tab 25		Shuffled c.p.i.					
ε	m	$C_{1,N}$ *N*(N-1)/2	$C_{m,N}$ *N*(N-1)/2	BDS	SD	$W_{m,N}$	d_m
0.13	2	449619	387089	0.12	0.07	1.73	0.15
0.13	4	449619	288325	0.34	0.19	1.85	0.29
0.13	6	449619	212739	0.32	0.27	1.17	0.44
0.13	8	449619	156115	0.23	0.33	0.72	0.59
0.13	10	449619	114191	0.15	0.35	0.43	0.75
0.07	2	332053	212774	0.16	0.10	1.60	0.34
0.07	4	332053	87059	0.18	0.16	1.14	0.68
0.07	6	332053	35031	0.08	0.13	0.64	1.02
0.07	8	332053	14114	0.04	0.09	0.45	1.36
0.07	10	332053	5713	0.02	0.05	0.38	1.70
0.04	2	208471	84076	0.08	0.07	1.10	0.56
0.04	4	208471	13326	0.02	0.04	0.36	1.12
0.04	6	208471	2053	0.00	0.01	-0.04	1.70
0.04	8	208471	319	0.00	0.00	-0.10	2.27
0.04	10	208471	55	0.00	0.00	0.24	2.81

MERIT-Infonomics Research Memorandum series
- 2001-

2001-001 **The Changing Nature of Pharmaceutical R&D - Opportunities for Asia?**
Jörg C. Mahlich and Thomas Roediger-Schluga

2001-002 **The Stringency of Environmental Regulation and the 'Porter Hypothesis'**
Thomas Roediger-Schluga

2001-003 **Tragedy of the Public Knowledge 'Commons'? Global Science, Intellectual Property and the Digital Technology Boomerang**
Paul A. David

2001-004 **Digital Technologies, Research Collaborations and the Extension of Protection for Intellectual Property in Science: Will Building 'Good Fences' Really Make 'Good Neighbors'?**
Paul A. David

2001-005 **Expert Systems: Aspects of and Limitations to the Codifiability of Knowledge**
Robin Cowan

2001-006 **Monopolistic Competition and Search Unemployment: A Pissarides-Dixit-Stiglitz model**
Thomas Ziesemer

2001-007 **Random walks and non-linear paths in macroeconomic time series: Some evidence and implications**
Franco Bevilacqua and Adriaan van Zon

2001-008 **Waves and Cycles: Explorations in the Pure Theory of Price for Fine Art**
Robin Cowan

2001-009 **Is the World Flat or Round? Mapping Changes in the Taste for Art**
Peter Swann

2001-010 **The Eclectic Paradigm in the Global Economy**
John Cantwell and Rajneesh Narula

2001-011 **R&D Collaboration by 'Stand-alone' SMEs: opportunities and limitations in the ICT sector**
Rajneesh Narula

2001-012 **R&D Collaboration by SMEs: new opportunities and limitations in the face of globalisation**
Rajneesh Narula

2001-013 **Mind the Gap - Building Profitable Community Based Businesses on the Internet**
Bernhard L. Krieger and Philipp S. Müller

2001-014 **The Technological Bias in the Establishment of a Technological Regime: the adoption and enforcement of early information processing technologies in US manufacturing, 1870-1930**
Andreas Reinstaller and Werner Hölzl

2001-015 **Retrieval of Service Descriptions using Structured Service Models**
Rudolf Müller and Stefan Müller

2001-016 **Auctions - the Big Winner Among Trading Mechanisms for the Internet Economy**
Rudolf Müller

2001-017 **Design and Evaluation of an Economic Experiment via the Internet**
Vital Anderhub, Rudolf Müller and Carsten Schmidt

2001-018 **What happens when agent T gets a computer?**
Lex Borghans and Bas ter Weel

2001-019 **Manager to go? Performance dips reconsidered with evidence from Dutch football**
Allard Bruinshoofd and Bas ter Weel

2001-020 **Computers, Skills and Wages**
Lex Borghans and Bas ter Weel

2001-021 **Knowledge Transfer and the Services Sector in the Context of the New Economy**
Robin Cowan, Luc Soete and Oxana Tchervonnaya

2001-022 **Stickiness of Commercial Virtual Communities**
Rita Walczuch, Marcel Verkuijlen, Bas Geus and Ursela Ronnen

2001-023 **Automatic ontology mapping for agent communication**
F. Wiesman, N. Roos and P. Vogt

2001-024 **Multi Agent Diagnosis: an analysis**
N. Roos, A. ten Teije, A. Bos and C. Witteveen

2001-025 **ICT as Technical Change in the Matching and Production Functions of a Pissarides-Dixit-Stiglitz model**
Thomas Ziesemer

2001-026 **Economic stagnation in Weimar Germany: A structuralist perspective**
Thorsten H. Block

2001-027 **Intellectual property rights in a knowledge-based economy**
Elad Harison

2001-028 **Protecting the digital endeavour: prospects for intellectual property rights in the information society**
Elad Harison

2001-029 **A Simple Endogenous Growth Model With Asymmetric Employment Opportunities by Skill**
Adriaan van Zon

2001-030 **The impact of education and mismatch on wages: The Netherlands, 1986 - 1998**
Joan Muysken and Jennifer Ruholl

2001-031 **The Workings of Scientific Communities**
Robin Cowan and Nicolas Jonard

2001-032 **An Endogenous Growth Model à la Romer with Embodied Energy-Saving Technological Change**
Adriaan van Zon and İ. Hakan Yetkiner

2001-033 **How Innovative are Canadian Firms Compared to Some European Firms? A Comparative Look at Innovation Surveys**
Pierre Mohnen and Pierre Therrien

2001-034 **On The Variance of Market Innovation with the Number of Firms**
Robin Cowan

2001-035 **Non linear dynamics in US macroeconomic time series**
Franco Bevilacqua

2001-036 **Multinational Firms, Regional Integration and Globalising Markets: Implications for Developing Countries**
Rajneesh Narula

2001-037 **CONTRACT PROLONGATION IN INNOVATION PRODUCTION AS A PRINCIPAL-AGENT PROBLEM WITH MORAL HAZARD**
Thomas Ziesemer

Papers can be purchased at a cost of EURO 7 or USD 7 per report at the following address:

MERIT - P.O. Box 616 - 6200 MD Maastricht - The Netherlands - Fax : -31-43-388 4905
(* Surcharge of EURO 7 or USD 7 for banking costs will be added for orders from abroad)

Subscription: the yearly rate for the MERIT-INFONOMICS Research Memorandum series is EURO 150 or USD 150 or,
papers can be downloaded from INTERNET:

http://meritbbs.unimaas.nl
http://www.infonomics.nl

email: secr-merit@merit.unimaas.nl