Towards a Theory of Technological Mismatch

2 - Economic Growth

September 1995

M.J. Oude Wansink
MERIT
P.O. Box 616
6200 MD Maastricht
Phone (0)43 883873
Fax (0)43 216518
E-Mail M.Oudewansink@AlgEc.RuLimburg.NL

Contents

1. Introduction.. 1

2. The Model of Lucas (1988).. 1

3. The Concept of Knowledge... 4

4. A Model with Endogenous Growth... 9

5. Stability Analysis and Transitional Dynamics................................ 15

6. Summary and Conclusions... 19

References.. 21

Abstract

In this paper, the relationship between technological change and the labour market is analysed using a growth model. Economic growth is generated by private investment in human capital, which is the heart of technological change. The model developed in this paper resembles the model of Lucas (1988), but differs in some important definitions. These definitions make it possible to combine a steady state equilibrium (constant and positive growth rates) with production functions for both the research and the educational sector which are not linear. This latter feature is an improvement on the new growth theory. However, the model generates a system of dynamic equations that can only be used for stability analysis when additional assumptions are made with respect to the endogenous variables.

Acknowledgement

In preparing this paper, I was kindly helped by Hans de Graaff, Hugo Hollanders, Hans van Meijl, Joan Muysken, Bart Verspagen, Thomas Ziesemer and Adriaan van Zon. I thank them for their remarks and improvements. Only the author remains responsible for the contents of this paper.

1. Introduction

This paper is part of a PhD research project on technological change and labour market mismatches. Here, I will try to investigate the impact of technological change on the labour market by means of growth theory. A growth model will be presented, in which economic progress is endogenized by means of a human capital production function. Although this model only marginally differs from the model of Lucas (1988), it exhibits a new treatment of human capital and knowledge within a growth model.

In section 2, I will start by examining the Lucas model of economic growth. After a desciption of the full model, I will pay only little attention to the steady state solutions to the model. The section mereley serves as both an introduction to Lucas-like modelling and as a reminder of the Lucas model itself.

In section 3, I will elaborate on the concepts of schooling, human capital and knowledge and the way they are incorporated in models of economic growth. For this purpose, I will elaborate on the theory of human capital, as originated by Schultz (1960) and Becker (1964), and the role of human capital in growth theory.

In section 4, I will present a model with endogenous economic growth, in which private investment in human capital is the growth generating motor of the economy (like in the Lucas model). Although my model and the Lucas model are structurally very similar, the former combines constant growth rates and marginal decreasing returns to human capital investments in the steady state, whereas the latter combines constant growth rates with constant returns to investments in human capital in the steady state.

In section 5, some conclusions will be drawn with respect to the process of transition and the characteristics of a steady state.

Section 6 contains a summary and conclusions.

2. The Model of Lucas (1988)

In his paper, Lucas (1988) focusses on the role of private investment in human capital in the process of technological change and economic growth.

He stresses the fact that the investment decision refers to the individual time allocation problem, because investing in human capital (schooling, on the job training) takes time. Time can be used for production purposes, raising consumption possibilities, and as a consequence, investments in human capital are not costless in the sense of opportunity costs (apart from the fact that in most countries, people must pay for acquiring education). Lucas' definition of human capital is simple:

> By an individual's 'human capital' I will mean [..] simply his general skill level, so that a worker with human capital *h(t)* is the productive equivalent of two workers with *½h(t)* each, or a half-time worker with *2h(t)*. (Lucas (1988), p.17)

Furthermore, Lucas distinguishes between non-leisure time devoted to production (of consumer goods), *u*, and non-leisure time devoted to the accumulation of individual human capital, *1-u*. The growth of human capital in period *t*, $\dot{h}(t)$, is related to the level of human capital in the following way:

(1) $$\dot{h}(t) = \delta[1-u(t)]h(t)^{\zeta}$$

where δ is a productivity parameter. In his model, Lucas assumes $\varsigma=1$, and (1) becomes linear in all variables. In the following section, this equation will be discussed at length.

The rest of the Lucas model ressembles a standard neoclassical model of growth. Infinitely living (representative) families maximize utility *U*, which depends positively on private consumption levels. Lucas uses the constant intertemporal elasticity of substitution (CIES) utility function (see Barro and Sala-i-Martin (1995), p.64):

(2) $$U(t) = \frac{N}{1-\sigma}(c^{1-\sigma}-1)$$

where *N* is the size of the (working) population, *c* is the level of (per capita) consumption and σ is the elasticity of substitution (see Barro and Sala-i-Martin (1995), p.64).

Consumer goods will be produced according to the following Cobb-Douglas production function:[1]

$$y = AK^{\beta}(uNh)^{1-\beta} \tag{3}$$

where A is an exogenous technology parameter and K is the level of (physical) capital inputs. The latter accumulate according to the following rule:

$$\dot{K} = y - c \tag{4}$$

In conclusion, infinitely living families maximize (2), subject to a budget constraint given in (4) and a human capital production function given in (1). The current value Hamiltonian of this problem becomes:

$$H = \frac{N}{1-\sigma}(c^{1-\sigma}-1) + \theta_1[AK^{\beta}(uNh)^{1-\beta}h^{\gamma} - Nc] + \theta_2[\delta h(1-u)] \tag{5}$$

The optimal control theory can be applied to solve for the steady state (constant) growth rate of human capital, $\dot{h}/h = \hat{h} = \nu$:

$$\nu = \sigma^{-1}(\delta - \rho + \lambda) \tag{6}$$

where ρ is a discount rate and $\lambda = \dot{N}/N = \hat{N}$ is the growth rate of the population N. It is obvious from (6) that the growth rate of human capital and therefore the growth rate of production can be positive even when the size of the population is constant ($\lambda = 0$), which was not the case in traditional neoclassical theories of growth.[2]

So far, I have described the structure of the Lucas model and presented its main steady state solution, which is the growth rate of human capital. In the following section, I will make some comments with respect to the role of human capital and knowledge in this and other endogenous growth models.

---

1. Lucas incorporates an external effect from the average level of human capital in the production function. I have omitted this externality, because it is not crucial for a comparison between the Lucas model and mine.

2. See Barro and Sala-i-Martin (1995), Chapter 2.

3. The Concept of Knowledge

Equation (1) is crucial to the model of Lucas. The assumption that $\varsigma=1$ forces the steady state growth rate of human capital to be constant, as is obvious from this equation. When $\varsigma<1$, the human capital growth rate converges to zero over time (and so does economic growth) and when $\varsigma>1$, the system explodes. However, Schultz (1960) and Becker (1964) have originated a human capital theory, in which the education decision of an individual is defined analogously to a standard investment decision. Investing in education and human capital by individuals continues over time until the expected returns from additional education (discounted higher wages) outweights the expected cost of additional education (discounted schooling outlays).

Indeed, the educational decision is a decision about time allocation, as was recognized by Lucas (p.17), but the time share devoted to the accumulation of human capital must decline over a life time as a result of the abovementioned investment decision by individuals.[3] This implication is crucial to the human capital theory, but is not incorporated in the model of Lucas. It was recognized by Lucas (p.19) that diminishing returns from investments in human capital are not described by (1) with $\varsigma=1$, but with $\varsigma<1$. However, as discussed above, $\varsigma<1$ will not result in constant steady state rates of growth in his model.

In addition to this remark, meaning of equation (1) has never been discussed clearly in the literature. When h is called "human capital", the statement "human capital is needed in order to produce new human capital" seems quite plausible. For example, someone first has to learn a language before he or she can read the literature written in that language. However, when h is labelled "labour efficiency", or "knowledge", the statement "labour efficiency is needed in order to produce new labour efficiency" seems quite strange in the presence of productivity parameter δ in (1). Furthermore, the statement "*all* knowledge is needed in order to produce new knowledge" highly exagerates the capacities of many scientists or inventors. In other words, the variables and their representation in equation (1) need some reconsideration.

3. I like to stress the importance of this point. In some cases, it can be observed that learning and schooling activities take a quite constant share of time in any period of a person's life. However, it can hardly be defended that knowledge acquired in the last periods of the working life of such a person will be paid back in terms of 'returns on investment'.

Let me start by defining human capital, labour efficiency and knowledge once again and use them for another representation of the problem. **Human capital** is treated as the individual stock of acquired education. During schooling time, individuals acquire part of the total stock of knowledge. Individual **labour efficiency** is treated analogously with human capital and it can achieve a maximum level of efficiency when all knowledge has been acquired (Muysken and Oude Wansink (1994)). This means that the maximum level of efficiency is determined by the stock of knowledge, which will not be constant over time. The **stock of knowledge** is defined as the total amount of (productive) information or techniques that is known at a certain moment in time. This stock refers to the set of knowledge of all individuals and consequently, it is hardly possible to any individual to possess the entire stock of knowledge. Only some well educated scientists may come close to it.

Let me give an illustration of how these concepts are used in relation to each other. In Oude Wansink (1995), I pointed at the obsolescence of skills, whereas in Muysken and Oude Wansink (1994), the concept of minimum and maximum efficiency levels was described. The integration of these concepts is illustrated in Figure 1.

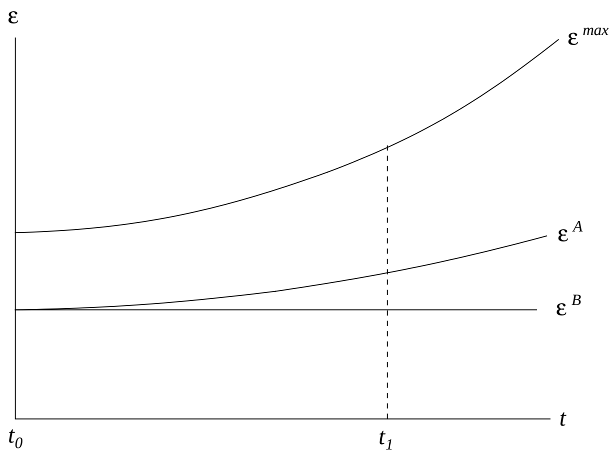

Figure 1
The Development of Efficiency Levels

Individual labour efficiency can only increase when investments in education are made.[4] Furthermore, there is a minimum required and a maximum obtainable level of efficiency, determined by the situation in the labour market and the rate of technological change, respectively. An increase in the maximum efficiency level ε^{max} leads to a situation in which new entrants in the labour market, fresh from their schools, have relatively higher efficiency levels than those holding the jobs. This is the result of technological change, which directly increases both the maximum obtainable level of efficiency and the efficiency of the education sector. The maximum level ε^{max} is referred to as the stock of knowledge, whereas the actual individual level of efficiency is referred to as labour efficiency or the *amount* of human capital acquired by an individual.

In Figure 1, individual A invested in human capital (schooling) between period t_0 and t_1, resulting in an increased efficiency level ε^A. Individual B did not invest in human capital (no schooling) in the same period and his level of efficiency ε^B remained constant. There are two reasons for the skills of individual B to become obsolete:

- efficiency level ε^A becomes the minimum level of efficiency demanded by firms in the labour market and individual B becomes unemployed;
- the remuneration for the supply of efficiency level B (the wage level) decreases below the minimum subsistence level and individual B is forced to look for additional income or another job.

When individuals invest in education and knowledge, their efficiency will increase but marginally decrease with the schooling effort made. Gradually, their efficiency level will approach the maximum attainable efficiency level ε^{max}. This is illustrated in Figure 2. The development of the efficiency level ε^i for individual i is depicted as a function of the number of schooling years s. The decision to raise the level of efficiency from ε^A up to ε^B depends on wages, the cost of schooling and a discount rate. The human capital investment decision rule can be defined as:

(7)
$$\int_A^B c_s(t)e^{-\rho t}dt \leq \int_B^T w(t)(\varepsilon^B - \varepsilon^A)e^{-\rho t}dt$$

where c_s is the cost of schooling, w is the wage rate per efficiency unit, T is the period of retirement and ρ is a discount rate.

4. On-the-job learning is not present in my analysis, but can be incorporated very easily.

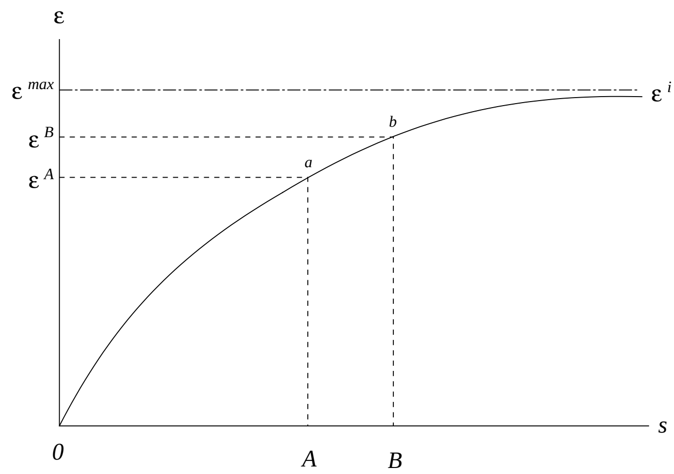

Figure 2
Efficiency Level and Schooling

In (7),the discounted cost of additional schooling on the left side must at least outweight the discounted *extra* wage after schooling for an individual to invest in the additional education. It is easy to see that the slope of a line tangent to point *b* on the ε^i curve in Figure 2 is equal to $\tilde{c}_s / \tilde{w}$, where the tilde on a variable stands for the discounted value of that variable. The higher the discounted schooling cost, the steeper the tangent line to the ε^i curve, the shorter the period of schooling. The higher the discounted wage per efficiency unit, the flatter the tangent line to the ε^i curve, the longer the period of schooling.

Now that I have described the concave relation between efficiency/human capital and schooling effort and the education investment decision corresponding to it, the development of the maximum obtainable level of efficiency ε^{max}, or better: the increase in the stock of knowledge, must be described. As an example of another definition of the growth of the stock of knowledge, I start with the equation of motion used in the Romer (1990) model:[5]

(8) $$\dot{A} = \delta[1 - u(t)]A$$

where A is an index for the stock of knowledge.

5. The variables have the same meaning as before. The human capital variable H_A in the Romer model is defined as the *1-u(t)* expression from the Lucas model.

It is the same equation used by Lucas (1988), but the state variable is called "knowledge" or "blueprints" by Romer instead of "human capital" by Lucas. Romer calls the *1-u(t)* term human capital, but is not very clear about the exact meaning of this human capital.[6] However, his idea that human capital is used as a production factor in the research sector is very appealing, because it explains why knowledge can be accumulated independently from the creation of human capital and the acquisition of knowledge by individuals. When human capital is assumed to exhibit increasing but marginally decreasing returns in the production of knowledge, this would lead to the following Cobb-Douglas production function of knowledge:

$$\dot{A} = (uh)^{\beta} \tag{9}$$

where u is the non-leisure time share devoted to research and h is a human capital index (β is a parameter with $0<\beta<1$). The same concave relation can be defined for the production of education (see next section). With these concave relations, the limited amounts of time devoted to research and education, as computed from the human capital decision rule in (7), can be confronted with the optimum constant fraction of time devoted to research and education as computed from a Lucas like model of economic growth.

In the model outlined in the next section, I will incorporate the abovementioned concave production functions in a model of economic growth. As before, I will assume the amount of human capital and the level of labour efficiency to be equivalent, for which I will use variable ε. The technology variable A, which is exogenous in the model of Lucas, will serve as the stock of knowledge in my model.

---

6. For instance, Romer assumes that "human capital H is a distinct measure of the cumulative effect of activities such as formal education and on-the-job training" and that it is the "rival component of knowledge" (p. S79). Furthermore, "any person can devote human capital to either the final-output sector or the research sector. Implicitly this formulation neglects the fact that L [labour services, MOW] and H are supplied jointly. To take the equations used here literally, one must imagine that there are some skilled persons who specialize in human capital accumulation and supply no labor" (p. S85). However, labour services L "are measured by counts of people" (p. S79), which means that human capital H must incorporate a part that is also "measured by counts of people". It means that the growth rate in his model would be dependent on some measure of the *number* of scientists, which is very unlikely.

4. A Model with Endogenous Growth

As mentioned before, I have developed a model with endogenous growth which ressembles the model of Lucas (1988) in many ways. Therefore, equations (2) and (4) will also be used in my model. The equation of motion (1) and the production function in (3) will be different, whereas another equation of motion will be added to the system.

The production function will be slightly different from the one in (3), because the non-leisure time devoted to production by workers must be corrected for the time devoted to research (u) and to schooling (v). The production function then becomes:

$$y = AK^{\alpha}[(1-u-v)\varepsilon]^{1-\alpha} - c \tag{10}$$

with $0<\alpha<1$. As described in the previous section, the acquisition (and therefore the growth) of human capital is concavely related to its own level, but directly influenced by the stock of knowledge A. This can be represented by the following equation:

$$\dot{\varepsilon} = (\varepsilon v)^{1-\beta}A \tag{11}$$

with $0<\beta<1$. Finally, equation (9) can be rewritten in terms of efficiency:

$$\dot{A} = (\varepsilon u)^{\beta} \tag{12}$$

The β parameter in (11) and (12) is the same parameter, indicating that there exists a trade-off between the productivity of scientists/teachers in the research and the education sector. This is caused by competition in the labour market.

The following current value Hamiltonian can be constructed given the intertemporal optimization problem of maximizing (2), subject to (4), (10), (11) and (12):

$$\mathrm{H} = \frac{1}{1-\sigma}(c^{1-\sigma}-1) + \theta_1\left[AK^{\alpha}[(1-u-v)\varepsilon]^{1-\alpha} - c\right] + \theta_2(\varepsilon u)^{\beta} + \theta_3(\varepsilon v)^{1-\beta}A \tag{13}$$

with state variables A, K and ε, costate variables θ_1, θ_2 and θ_3 and control variables c, u and v.

The first order conditions give rise to the following system of equations:

(14) $$c^{-\sigma} - \theta_1 = 0$$

(15) $$-\theta_1(1-\alpha)AK^{\alpha}\varepsilon^{1-\alpha}(1-u-v)^{-\alpha} + \theta_2\beta\varepsilon^{\beta}u^{\beta-1} = 0$$

(16) $$-\theta_1(1-\alpha)AK^{\alpha}\varepsilon^{1-\alpha}(1-u-v)^{-\alpha} + \theta_3(1-\beta)\varepsilon^{1-\beta}u^{-\beta}A = 0$$

(17) $$\dot{\theta}_1 = \rho\theta_1 - \theta_1\alpha AK^{\alpha-1}\varepsilon^{1-\alpha}(1-u-v)^{1-\alpha}$$

(18) $$\dot{\theta}_2 = \rho\theta_2 - \theta_1K^{\alpha}\varepsilon^{1-\alpha}(1-u-v)^{1-\alpha} - \theta_3(\varepsilon v)^{1-\beta}$$

(19) $$\dot{\theta}_3 = \rho\theta_3 - \theta_1(1-\alpha)AK^{\alpha}\varepsilon^{-\alpha}(1-u-v)^{1-\alpha} - \theta_2\beta\varepsilon^{\beta-1}u^{\beta} - \theta_3(1-\beta)\varepsilon^{-\beta}v^{1-\beta}A$$

Together with (4), (11) and (12) 9 equations are available in order to get a steady state solution for 9 variables mentioned above. By definition, the growth rates of c, K, A and ε are constant in a steady state and those of v and u are assumed to be equal to zero (time shares devoted to any activity are constant). Following the analysis of Lucas, it is easy to find that:

(20) $$\hat{c} = \hat{K} = \left[\frac{1-\alpha+\beta}{1-\alpha}\right]\hat{\varepsilon}$$

Furthermore, dividing both sides of (12) by A and differentiating yields:

(21) $$\hat{A} = \beta\hat{\varepsilon}$$

Expressing the first order conditions in growth rates yields:

(22) $$\hat{\theta}_1 = -\sigma\hat{c}$$

(23) $$\hat{\theta}_1 + \hat{A} + \alpha\hat{K} + (1-\alpha)\hat{\varepsilon} = \hat{\theta}_2 + \beta\hat{\varepsilon}$$

(24) $$\hat{\theta}_1 + \hat{A} + \alpha\hat{K} + (1-\alpha)\hat{\varepsilon} = \hat{\theta}_3 + (1-\beta)\hat{\varepsilon} + \hat{A}$$

(25) $$\hat{\theta}_1 = \rho - \alpha A K^{\alpha-1}\varepsilon^{1-\alpha}(1-u-v)^{1-\alpha}$$

(26) $$\hat{\theta}_2 = \rho - A^{-1}(\varepsilon u)^{\beta}\left[\frac{\beta}{1-\alpha}\,\frac{1-u-v}{u} - \frac{\beta}{1-\beta}\,\frac{v}{u}\right]$$

(27) $$\hat{\theta}_3 = \rho - (1-\beta)(\varepsilon v)^{-\beta}A$$

Growth rates of the equations of motion (4), (11) and (12) are given by:

(28) $$\hat{K} = AK^{\alpha-1}\varepsilon^{1-\alpha}(1-u-v)^{1-\alpha} - cK^{-1}$$

(29) $$\hat{A} = (\varepsilon u)^{\beta}A^{-1}$$

(30) $$\hat{\varepsilon} = \varepsilon^{-\beta}v^{1-\beta}A$$

From (26), (29) and (21) it is clear that:

(31) $$\hat{\theta}_2 = \rho - \beta\hat{\varepsilon}\left[\frac{\beta}{1-\alpha}\,\frac{1-u-v}{u} - \frac{\beta}{1-\beta}\,\frac{v}{u}\right]$$

From (27) and (30) it can be found that:

(32) $$\hat{\theta}_3 = \rho - (1-\beta)v^{-1}\hat{\varepsilon}$$

Substitute (20), (22), (29) and (32) in (23), dividing by $\hat{\varepsilon}$ and rearranging yields

(33) $$\hat{\varepsilon} = \frac{(1-\alpha)\rho v}{(1-\alpha)(1-\beta) + \left[\beta - \sigma(1-\alpha+\beta)\right]v}$$

This equation can be used to express the growth rates of c, K and A, as defined in (20) and (21) in terms of u and v. The last step is to find the solutions for these latter mentioned variables. Equations (20), (21), (22) and (31) can be used in order to get the following expression for $\hat{\varepsilon}$ from (23):

$$\hat{\varepsilon} = \frac{\rho}{\left[\frac{(\alpha-\sigma)(1-\alpha+\beta)}{1-\alpha} + 1-\alpha+\beta\left(\frac{\beta}{1-\alpha}\frac{1-u-v}{u} - \frac{\beta}{1-\beta}\frac{v}{u}\right)\right]} \tag{34}$$

Equation (34) must equal (33) and from this equality, I can get the following expression for *u*:

$$u = \frac{-\beta^2\left[v(1-\beta)-v^2(2-\beta-\alpha)\right]}{(1-\alpha)(1-\beta)(\beta-1) + \left[(1-\alpha)(1-2\beta)+\beta^2(\beta-\alpha)\right]v} \tag{35}$$

Next, substituting (22) and (28) in (25) yields:

$$-\sigma\hat{c} = \rho - \alpha(\hat{K} + \frac{c}{K}) \tag{36}$$

Using (20) and (33), I can get the following expression from (36) that contains *v*, *c* and *K* as endogenous variables:

$$\rho - \left[\frac{(\alpha-\sigma)(1-\alpha+\beta)\rho v}{(1-\alpha)(1-\beta)+\left[\beta-\sigma(1-\alpha+\beta)\right]v}\right] - \alpha\frac{c}{K} = 0 \tag{37}$$

The growth rates of consumption *c* and capital inputs *K* are the same by (20), which means that the ratio *c/K* is constant over time. Therefore, I introduce the steady state constant *x=c/K*. From (37), a definition of *v* can then be obtained:

$$v = \frac{(\rho-\alpha x)(1-\alpha)(1-\beta)}{\rho(\alpha-\beta)(1-\alpha) + \alpha x\left[\beta-\sigma(1-\alpha+\beta)\right]} \tag{38}$$

Substitution of (38) in (35) gives a solution for *u*:

$$u = \frac{-\beta^2(\alpha x-\rho)\left[\alpha x(\alpha^2+\alpha(\beta+\sigma-3)-\beta\sigma-\sigma+2)-2\rho(\alpha-1)^2\right]}{\left[\alpha x(\alpha\sigma+\beta(1-\sigma)-\sigma)-\rho(\alpha-\beta)(\alpha-1)\right]\left[\alpha x(\beta+\sigma-1)-\rho(\alpha+\beta-1)\right](\alpha-\beta-1)} \tag{39}$$

I will now turn to the analysis of the steady state solutions in order to find out under which conditions the solution is feasible or not. The solution for *v* in (38) must be positive.

Because *(1-*α*)* and *(1-*β*)* are positive by definition, 3 criteria determine the sign of *v* in (38):

- $\rho <> \alpha$?
- $\alpha <> \beta$?
- $\sigma <> \beta/(1-\alpha+\beta)$?

In Table 1, I put these criteria on the axes and describe the conclusion with respect to the sign of *v*. Furthermore, *v* must also be smaller than 1. Assuming σ to be positive, this restriction leads to the following condition:

(40)
$$\rho < \frac{\alpha x}{1-\alpha}$$

Given the predefined contraint of $0<\alpha<1$, all possibilities presented in Table 1 are feasible when taking into account the condition in (40).

The Sign of v

$[1] = \left|\rho(\alpha-\beta)(1-\alpha)\right|$ and $[2] = \left|\alpha x\left[\beta-\sigma(1-\alpha+\beta)\right]\right|$

	$\rho > \alpha x$	$\rho < \alpha x$
$\alpha > \beta$ $\sigma < \beta/(1-\alpha+\beta)$	$v > 0$	$v < 0$
$\alpha > \beta$ $\sigma > \beta/(1-\alpha+\beta)$	$v > 0$ if [1] > [2]	$v > 0$ if [1] < [2]
$\alpha < \beta$ $\sigma < \beta/(1-\alpha+\beta)$	$v > 0$ if [1] < [2]	$v > 0$ if [1] > [2]
$\alpha < \beta$ $\sigma > \beta/(1-\alpha+\beta)$	$v < 0$	$v > 0$

Table 1

In conclusion, I have 6 possible situations reported in Table 1 in which *v* can take a value between 0 and 1.

The result for *u* in (39) is more complex to handle, but from (35), assuming *v* to be positive, 3 criteria can be developped in order to test the sign of *u*.

Again, a table can be constructed with 8 possible situations. The criteria give rise to 7 feasible solutions, for which the conditions are reported in Table 2. As before, I could find some additional criteria from the restriction that $u<1$, but it only possible to calculate some restriction on the value of v from (35). An interpretation of this restrictions can hardly be given and in some respect, such an explanation ressembles the conclusions that can be drawn from Table 2. However, this restriction does not change any of the criteria presented in Table 2, which means that 7 possible different combinations of α-, β- and v-value ranges can be associated with a positive value of u.

The Sign of u

$[1] = |(1-\alpha)(1-2\beta)|,\ [2] = |\beta^2(\beta-\alpha)|,$

$[3] = |(1-\alpha)(1-\beta)(\beta-1)|$ and $[4] = |[(1-\alpha)(1-2\beta)+\beta^2(\beta-\alpha)]v|$

	$v < \frac{1-\beta}{2-\alpha-\beta}$	$v > \frac{1-\beta}{2-\alpha-\beta}$
$\beta > 1/2$ $\alpha > \beta$	$u > 0$	$u < 0$
$\beta > 1/2$ $\alpha < \beta$	$u > 0$ if [1] > [2] or [3] > [4]	$u > 0$ if [1] < [2] and [3] < [4]
$\beta < 1/2$ $\alpha > \beta$	$u > 0$ if [1] < [2] or [3] > [4]	$u > 0$ if [1] > [2] and [3] < [4]
$\beta < 1/2$ $\alpha < \beta$	$u > 0$ if [3] > [4]	$u > 0$ if [3] < [4]

Table 2

Finally, the value of $\hat{\varepsilon}$ should be positive in a steady state in order to have $\hat{c}$, $\hat{K}$ and $\hat{A}$ positive as well (from (20) and (21), respectively).

From (33) it can be found that $\hat{\varepsilon} > 0$ imposes the following restriction on σ:
The right hand side of (41) is always positive, which means that for any value of σ and a positive v, the model predicts a positive growth rate of efficiency ε.

(41) $$0 < \sigma < \frac{(1-\alpha)(1-\beta)+\beta v}{(1-\alpha+\beta)v}$$

Note that the range defined in (41) does not exclude any of the possible outcomes for v presented in Table 1.

In the next section, I will focus on the stability characteristic of a steady state. It will be interesting to see how the results relate to the theoretical issues raised in section 3. Besides, the dynamic behaviour of the model will be analysed.

5. Stability Analysis and Transitional Dynamics

The system of first order conditions, given in (4), (11)-(12) and (14)-(19) contains 3 control variables, 3 state variables and 3 costate variables. The costate variables can be eliminated by means of substitution, but a very complex system of equations still remains to be solved. Therefore, conclusions with respect to the stability of the system and its transitional dynamics can only be drawn when additional assumptions are made. In this section, two different scenarios will be of the model outlined in the former section will be described.

The first scenario describes the individual adjustment of time when growth rates of variables and shadow prices are constant in the process of transition towards a steady state equilibrium. In other words, when $\hat{\theta}_2$ and $\hat{\varepsilon}$ are assumed to be constant in the transition process, (31) can be restated as:

(42) $$v = \phi_1 - \phi_2 u$$

where

(43) $$\phi_1 = \frac{1-\beta}{2-\alpha-\beta}$$

and

(44) $$\phi_2 = \frac{(1-\alpha)(1-\beta)(\rho-\hat{\theta}_2)+(1-\beta)\beta^2\hat{\varepsilon}}{(2-\alpha-\beta)\beta^2\hat{\varepsilon}}$$

The slope of the line defined in (42) depends on the level of ρ in relation to $\hat{\theta}_2$. I assume a trade-off between time spent on research and time spent on education, which means that (42) describes a downward sloping curve in the (u,v) space.[7] Next, the additional assumption of $\hat{\theta}_3$ to be constant over time, implies that (32) can be expressed as:

(45) $$v = \phi_3$$

where

(46) $$\phi_3 = \frac{(1-\beta)\hat{\varepsilon}}{\rho-\hat{\theta}_3}$$

which is constant. The sign of the right hand side of (46) depends on the level of ρ in relation to $\hat{\theta}_3$, but only positive values for v are meaningful.

In Figure 3, I have depicted the situation described by (42) and (45). The available amount of time has been normalized to unity. The downward sloping curve described by (42) corresponds to the line *AB* in Figure 3, indicating that when $u=0$, v is positive in point *A* (=ϕ_1). The constant value for v described by (45) corresponds to the horizontal line at $v=v^*$. It is clear from the figure that the steady state equilibrium (u^*,v^*) is known when the initial (constant) value for v is known by (45). This is a once-and-for-ever adjustment within one period (for instance, from initial point $u=C$ to $u=u^*$), which means that there are no transitional dynamics. Note that the constant v defined by (45) is restricted to be less than unity, because working time (1-u-v) must initially be positive. Finally, in Figure 3, the white area bordered by *A* and *B* contain possible steady state values for u and v, whereas the shaded area represents the remaining possible level of the steady state working time (1-u-v).

7. This assumption relates to the time allocation problem as discussed upon in section 3.

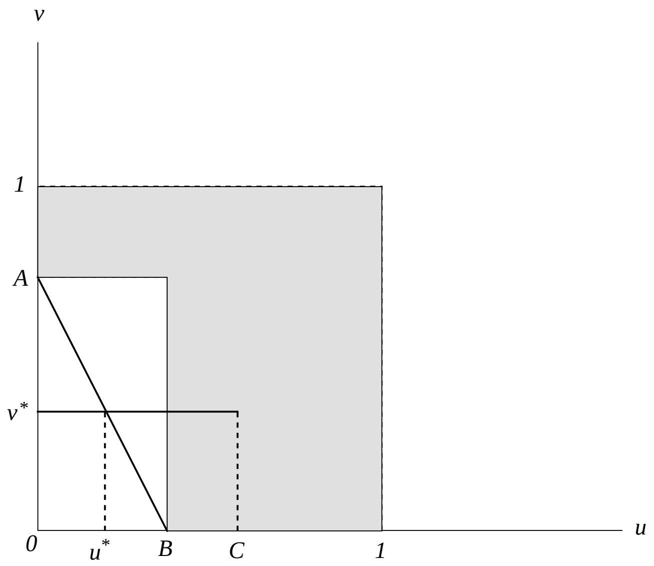

Figure 3
Equilibrium in Time Allocation

The second scenario refers to the individual adjustment of consumption and saving plans, given the constancy of growth rates of technological change and shadow prices and given fixed time shares devoted to the different activities. Substituting (25) in (28) and rearranging yields:

(47) $$\hat{K} = \left[\frac{\rho - \hat{\theta}_1}{\alpha}\right] - \frac{c}{K}$$

The first term of the right hand side of (47) is assumed to be constant, which means that differentiating (47) gives the following expression:

(48) $$\partial \hat{K} / \partial t = \frac{c}{K}\,[\hat{c} - \hat{K}]$$

Next, substituting (22) in (23) and rearranging yields:

(49) $$\hat{c} = \frac{\alpha}{\sigma}\,\hat{K} + \left[\frac{(1 - \alpha - \beta)\hat{\varepsilon} + \hat{A} - \hat{\theta}_2}{\sigma}\right]$$

The second term of the right hand side of (49) is assumed to be constant, which means that differentiating (49) gives the following expression:

(50) $$\partial \hat{c} / \partial t = \frac{\alpha}{\sigma} (\partial \hat{K} / \partial t)$$

Both (48) and (50) can be used to create a phase diagram, in which the stability of a steady state solution can be analysed.[8] Setting $\partial \hat{K} / \partial t = 0$ in (48) defines the linear relation $\hat{c} = \hat{K}$. This relation could have been expected to show up in the stability analysis since the growth rates of capital K and consumption c are the same in a steady state by (20). Setting $\partial \hat{c} / \partial t = 0$ in (50) defines $\hat{K}$ to be constant. In Figure 4, I have depicted this situation in a phase diagram, together with the dynamic paths of both variables as given by (48) and (50). Note, that the direction of $\partial \hat{c} / \partial t$ in (50) is determined by the direction of $\partial \hat{K} / \partial t$ in (48).

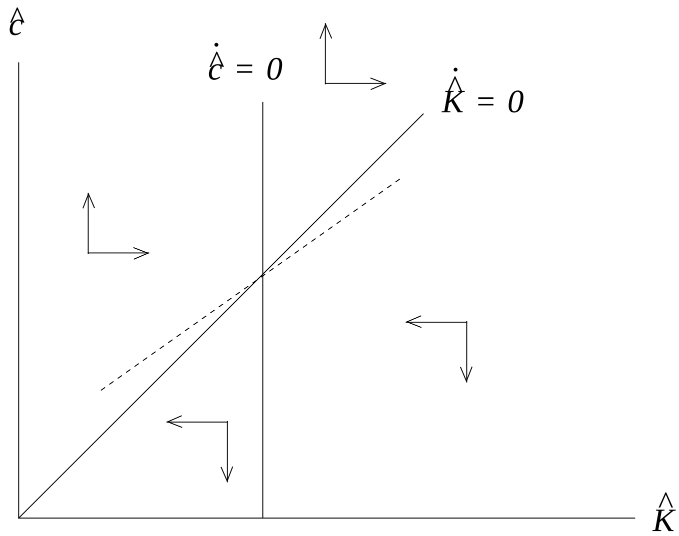

Figure 4
Dynamics of Consumption and Capital

The intersection of both lines in Figure 4 represent the steady state equilibrium, which is a saddle point.

8. See Chiang (1984) and Chiang (1992) for the use of phase diagrams and other tools of dynamic analysis.

This can be verified by defining the Jacobian matrix of the dynamic equations (48) and (50):

$$J = \begin{bmatrix} -\dfrac{c}{K} & \dfrac{c}{K} \\ \dfrac{\alpha}{\sigma} & 0 \end{bmatrix} \tag{51}$$

The determinant of the Jacobian matrix is equal to $-\alpha c/\sigma K$ which is negative. This is an indication that the steady state equilibrium is a saddle point. Therefore, as depicted in the Figure by the dashed line, there exists a unique path towards the steady state equilibrium.

In conclusion, the assumption of constant growth rates of technology variables and shadow prices, as well as fixed time shares *u* and *v* leads to a dynamic adjustment of consumption and saving patterns as depicted in Figure 4. The resulting steady state equilibrium is a saddle point.

This completes the discussion of the stability and the transitional dynamics of the model. The last section will be used to summarize the main findings of this paper.

6. Summary and Conclusions

In the model of Lucas (1988), private investment in human capital is the growth generating motor of the economy. The investment decision in his model is a time allocation problem with respect to schooling and working time. The creation of human capital, and therefore economic progress,is endogenized in his model. The Lucas model is characterized by a steady state equilibrium in which growth rates are constant and positive, even when population growth is absent, and non-leisure time shares devoted to schooling and working are constant too.

The production function of human capital in the Lucas model is linear in the stock of acquired human capital. This means that the marginal productivity of the human capital creating sector is constant with respect to both time and the amount of acquired human capital.

This is not acceptable when confronted with the human capital theory as originated by Schultz (1960) and Becker (1964), because constant marginal returns to education limits the amount of education to be acquired only by time and not by the inputs of the educational production process. Therefore, in a model where individuals (or families) live infinitly, this linearity cannot give a sound explanation for the human capital investment decision by individuals.

In the model presented in this paper, a distinction has been made between time devoted to work, schooling and research activities. In all sector, producers are confronted with a production function that exhibits increasing but marginally decreasing returns to scale. One exception is formed by the use of the available stock of knowledge in the education sector. The model can be solved when it comes to the identification of a steady state equilibrium in which all growth rates of variables and time shares are constant, but an autonomous differential system with 6 endogenous variables remains to be solved in order to reach at conclusions with respect to the stability and the transitional dynamics of the model.

Two different scenarios have been investigated in this paper. In the first scenario, individuals are faced with the adjustment of time shares devoted to work, schooling and research activities. The growth rates of technological and behavioural variables, as well as those of shadow prices are assumed to be constant during the period of transition. A simple linear system of equations remains to be solved, in which transitional dynamics are absent. The resulting steady state equilibrium is stable.

The second scenario describes how individuals are faced with the adjustment of consumption and saving plans. In this scenario, growth rates of technological variables and shadow prices are assumed to be constant, as well as the time shares. A dynamic system of equations remains to be solved, which can be done qualitatively by the construction of a phase diagram. The resulting steady state equilibrium is a saddle point.

The main contribution of this paper is the connection made between the new growth theory and the human capital theory. The production function of human capital, as used in this paper, can be used in standard applications of the human capital theory in order to conclude upon the contribution of education to economic growth, the productive efficiency of the educational sector, the optimal length of schooling spells and the optimal level of the cost of schooling.

References

- Barro, R.J., Sala-i-Martin, X. (1995), Economic Growth, McGraw-Hill, New York.
- Becker, G.S. (1964), Human capital, National Bureau of Economic Research, New York.
- Chiang, A.C. (1984), Fundamental Methods of Mathematical Economics, McGraw-Hill, Singapore, Third Edition.
- Chiang, A.C. (1992), Elements of Dynamic Optimization, McGraw-Hill, Singapore.
- Lucas, R.E. (1988), "On the Mechanics of Economic Development", Journal of Monetary Economics, vol. 22, pp. 3-42.
- Muysken, J., Oude Wansink, M.J. (1994), "Education, Efficiency and Economic Growth", paper, prepared for the PhD seminar on Economics of Technological and Institutional Change, Strassbourg, September 30 - October 5.
- Oude Wansink, M.J. (1995), "Towards a Theory of Technological Mismatch: 1 - Consumption", MERIT Research Memorandum, 2/95-015.
- Romer, P. (1990), "Endogenous Technological Change", Journal of Political Economy, vol. 98(5), pp. S71-S102.
- Schultz, T.W. (1960), "Capital Formation by Education", Journal of Political Economy, vol. 68(6), pp. 545-557.