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Health as a Principal Determinant

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by

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Abstract

We construct an endogenous growth model based on Lucas (1988) and van Zon and Muysken (2001). The model shows how the provision of health services influences an economy's rate of growth. Since growth is produced using labour services, increasing health activities also implies lowering output and possibly growth. At the same time a good health is necessary to be productive at all. Hence there is a direct trade-off between the health state of the population and growth performance. We add a stylized demographical and epidemiological module to the Lucas model that enables us to distinguish between care activities and cure activities. We illustrate how changes in the rate of morbidity and the rate of mortality influence the optimum allocation of scarce (labour-) resources over its various uses that include care-, cure-, final output production and human capital accumulation activities. We show that in this setting the relation between growth performance and health activities is parabolic. For low levels of health activities growth increases with the size of the health sector, and then starts falling again for further increases in the size of the health sector. Thus we illustrate that an increase in longevity as experienced by Western economies entails lower growth prospects for these economies, while for economies in the development stage, a positive relation between health activities and growth is more likely to exist. For the latter countries a double health dividend may be realized from increasing health care spending, i.e. the population can attain both a higher health level and a higher growth rate of income and consumption per head. For Western economies such a double dividend might still be obtained by increasing the productivity of the health sector rather than by expanding health activities, *ceteris paribus*. The main conclusion of our model is, however, that human capital accumulation is still the ultimate source of growth, but the health sector is instrumental in realizing the potential of the productive human capital embodied in the population. Because of that, an exclusive focus on cutting current health costs may interfere with the growth prospects of an economy, thus potentially tightening intertemporal macro economic budget constraints rather than loosening them.

JEL codes: I12, I18, O40, O41

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1. Introduction

For a long time economists have tended to ignore health as a relevant factor of production and important determinant of economic growth. The widely observed positive relationship between health expenditures and economic growth was considered the result of a strong positive income effect. However, gradually more and more economists have come to recognise that the relationship between health and economic growth is not only demand driven, but that health is also an important determinant of economic growth. The latter has mainly been recognised on the basis of empirical cross-country studies, starting for developing economies (see Strauss and Thomas, 1998, for an overview) and later also covering Western economies (Knowles and Owen, 1995; Barghava *cs.*, 2001; McDonald and Roberts, 2002; Webber, 2002). However, until this day only few attempts have been made to present a coherent account of the causal links between health and economic growth.

A pioneering analytical study in this field is Grossman (1972), followed up by, for instance, Muurinen (1982), Forster (1989), Ehrlich and Chuma (1990), Johansson and Lofgren (1995) and Meltzer (1997). However, these studies focus on the provision of health services from a micro economic demand perspective and ignore the positive productivity feedbacks of population health at the macro-level as an additional argument in favour of the provision of health services. In addition, this line of research does not recognise the possible interaction between health and the process of knowledge accumulation as the driving force behind economic growth.

Our model recognises explicitly that economic growth is driven by knowledge accumulation in the tradition of Lucas (1988), and as such is based on labour services supplied by healthy persons. The health state of the population at the aggregate level (i.e. the share of healthy people in the population) determines the extent to which potential labour

services embodied in the population can be used effectively. Moreover, knowledge accumulation also requires the spending of ‘healthy hours’, leading to the embodiment of knowledge in individual people. These two positive effects of health on economic performance are also recognised explicitly by Bloom and Canning (2000), who in addition mention two other impacts: improvements in longevity will increase savings (for retirement) and hence facilitate investment, and the occurrence of a “demographic dividend” because of the decline in infant mortality which creates an increase in the population of working age. Since we want to use our model to focus on developments in the West, this demographic dividend is less relevant. Instead, we recognise explicitly that an ageing population may become a drag on the economy, because aged persons do require increasing amounts of care from the health sector, whereas they have ceased to be productive.¹

Apart from care activities, a significant part of health activities take the form of cure in hospitals, by general practitioners, prevention activities, and so on. The important distinction between cure and care activities from a modelling point of view is that cure activities may change the health state of the population, while care activities do not. Because both activities take up scarce resources, whereas their impact on the population is principally different, we explicitly distinguish between both types of activities in our model.

A complicating factor is that at the aggregate level health production, both cure and care, takes place under decreasing returns to scale. Baumol (1967), for instance, takes the health sector as an example of a sector which permits ‘... only sporadic increases in productivity’ because ‘... there is no substitute for the personal attention of a physician...’, as opposed to human capital accumulation activities, which give rise to ‘... technologically progressive activities in which innovations, capital accumulation, and economies of large

¹ While Bloom and Canning (2000, n.1) argue that “Eventually, the large-sized cohorts will pass through the age distribution as surely as a pig that has been swallowed by a python”, we focus on the problem what happens to the Western python if the pig keeps growing on its way through.

scale make for a cumulative rise in output per man hour'.² In terms of our growth model this implies that we assume that the generation of health services is characterised by decreasing returns, whereas human capital accumulation generally is modelled using increasing returns.³

The increase in demand for both cure and care, together with the declining productivity in the provision of cure and care explain the dramatic increase in health costs over the past decades. Table 1 shows that in most Western countries health expenditures more than doubled their share in GDP during the past 40 years. Moreover, there is a fall in the productive base of the economy due to the ageing population in the West, as one can see from the increase in the share of the population aged 65 and over in Table 1.⁴ These developments are the cause of our strong interest in the relationship between health and economic growth.

It follows from the observations above, that our model should recognise the following principles: (1) health care and cure both use and produce resources for economic growth, directly with respect to goods production, indirectly with respect to knowledge production; (2) any activity depending on the input of labour hours will be negatively affected by a decrease in the health state of the population. Cure activities therefore regulate the level at which activities can be performed, including human capital accumulation activities; (3) cure and care activities are substitutes at the macro level, in that a higher level of cure activity reduces the share of non-healthy people in the population, thus reducing their need for care.

² Baumol 1967, pp. 416, 423, 415, respectively. It is these differences in productivity that are the cause of Baumol's disease.

³ Decreasing returns in health services are used in Forster (1989), Ehrlich and Chuma (1990) and Johansson and Lofgren (1995), increasing returns in human capital generation appear in the growth models mentioned above.

⁴ This picture is enforced by the projections presented in Jacobzone cs (2000), who show that for the OECD the age dependency ratio (percentage of persons aged 65 and over as a percentage of the working-age population) increased from 14.1 in 1960 to 20.6 in 2000 and will further increase to 32.7 in 2030.

	Population aged 65 years					
	Total expenditure on health, and over - % of total					
	% GDP			population		
	1960	1990	2000	1960	1990	2000
Australia	4,1	7,8	8,9	8,5	11,1	12,4
France		8,6	9,3	11,6	14	16,1
Germany	6,2 ^a	8,5	10,6			16,4
Italy		8	8,2	9,3	14,9	18,1
Japan	3	5,9	7,6	5,7	12,1	17,4
United Kingdom	4,5 ^a	6	7,3	11,7	15,7	15,9
United States	5	11,9	13,1	9,2	12,5	12

Source: OECD health statistics. (^a Figure for 1970)

Table 1 Health expenditure and ageing population in selected

Western economies, 1960 –2000

In a previous paper (van Zon and Muysken, 2001), we did not distinguish between cure and care activities, whereas we had a fixed population by assumption, forcing us to drop demographical issues from the outset. In this paper, however, we want to see what the implications of changes in demographical and epidemiological parameters, like the rates of mortality and morbidity, are likely to be for long term steady state growth. To this end we slightly depart here from the notion of a health index used in van Zon and Muysken (2001), that ‘corrects’ the effective supply of labour measured in efficiency units, by now using as health index the fraction of the population that is healthy enough to provide productive labour

services. By implication, the complement of this healthy sub-population is the part of the total population that can not provide these services because of ill health and/or old age. This enables us to introduce population dynamics into the Lucas framework, resulting from endogenous decisions regarding the provision of health services, next to exogenous developments in mortality and morbidity. Since the standard intertemporal utility function used in endogenous growth theory contains the number of heads as a positive argument next to consumption per head, and since the health state of the population influences the net growth rate of the population, a link between health activities and the population growth rate then provides a direct link between health activities and growth.

Apart from the direct link between health and growth decisions, there is an indirect link which is similar to the impact of education on economic growth. Investments in health, together with investments in education, determine the number of effective labour-services relative to the physical units of labour available that represent potential labour services. The resulting flow of labour services then has to be distributed over the various productive uses of available 'healthy' time, i.e. the production of final output, the rendering of health services in order to maintain the health state of the population, and the accumulation of productive knowledge. By regulating the provision of health and education services, one is able to influence the choice between consumption now and consumption in the future and hence the (steady state) growth rate of the economy.

In order to be able to present our health and endogenous growth model, we first reiterate the most important features of the Lucas model in the following section. To introduce endogenous population growth, we have constructed a very simply and stylised epidemiological and demographical module that is described in section 3. Section 4 then shows how the results obtained from this module are integrated with the original Lucas model and presents the formal results obtained for the modified model. Section 5 illustrates these

results through a numerical analysis that we have performed in order to show how growth and the allocation of health care resources react to changes in the system parameters. Then, in section 6, we show what the growth and health implications of this model are for Western economies. Finally, section 7 contains some concluding remarks.

2. The Lucas growth model revisited

We focus on the Lucas model without spill-overs from individual knowledge accumulation to the macro-level, because these spill-overs only strengthen the role of knowledge accumulation for endogenous growth, rather than being a condition sine qua non for growth. The Lucas model then consists basically of a standard intertemporal utility function, a standard neo-classical Cobb-Douglas production function exhibiting labour augmenting technical change, and a production function for the efficiency of labour that is driven by the accumulation of knowledge. The latter process takes time that can not be used to produce current output, but it leads to additional future output due to the higher efficiency of labour. This represents an intertemporal trade-off between consumption possibilities now and in the future, which Lucas solves through an intertemporal utility maximisation approach in the form of an optimum control problem.

The original Lucas model can now be summarised as follows:

$$Y = A \cdot ((1 - w) \cdot e \cdot P)^\alpha \cdot K^{1-\alpha} \quad (1)$$

In equation (1), Y represents output, A is a (constant) productivity parameter, P is the size of the population, K is the capital stock and e is the average efficiency per worker, while 1-w is the fraction of labour time allocated to final output production. Consequently, w is the

fraction of time per worker allocated to knowledge accumulation. Finally, $1 - \alpha$ is the partial output elasticity of capital. A and α are fixed parameters, w will be a fixed variable in the steady state, while e , P , Y and K will be growing in the steady state at fixed proportional rates of growth.

Savings, i.e. final output not consumed, are invested, and disregarding depreciation, the capital stock will accumulate in accordance with:

$$dK / dt = Y - c \cdot P \quad (2)$$

where c is consumption per head, and t represents time.

The production function for ‘new’ productive knowledge that manifests itself in the form of efficiency increases of labour, is given in equation (3) below, where δ_e is a fixed parameter that measures the productivity of the learning process:

$$de / dt = \delta_e \cdot e \cdot w \quad (3)$$

It should be noted that (3) implies that a constant allocation of labour time w , implies in turn a constant growth rate of efficiency per worker e . It is obvious that a higher value of w will result in more consumption (possibilities) in the future and less consumption now.

Lucas models the trade-off between present and future consumption by a standard Constant Intertemporal Elasticity of Substitution utility function:

$$U = \int_0^{\infty} e^{-\rho t} \cdot P \cdot (c^{1-\theta} - 1) / (1 - \theta) dt \quad (4)$$

In equation (4), ρ is the rate of discount and $1/\theta$ is the intertemporal elasticity of substitution. Note that now $\rho - \hat{P}$ acts as a kind of effective discount rate for the flow of utility coming out of the development over time of consumption per head.⁵

In the optimum w is chosen to maximize intertemporal utility U , subject to equations (1) – (3). The solution of the optimal control problem is then given by:⁶

$$e = \frac{\delta_e + \hat{P} - \rho}{\theta} \quad (5)$$

$$w = \frac{\delta_e + \hat{P} - \rho}{\delta_e \cdot \theta} \quad (6)$$

The steady state growth rate of consumption, output and capital per head in this Lucas economy is given by the growth rate of the efficiency of labour defined in (5). This economy grows fast if the productivity of the learning process is relatively high, and/or if the intertemporal elasticity of substitution is high. It grows slower if the effective rate of discount $\rho - \hat{P}$ rises, as one would expect, since the benefits from postponing consumption now take the form of increased future consumption that is valued less if the rate of discount rises. The allocation of time to knowledge accumulation w rises with an increase in the productivity of the learning process and with an increase of the intertemporal elasticity of substitution. In both cases it becomes more profitable (utility-wise) to invest more in future consumption; in the former case because the returns to investment have gone up, and in the latter case because the valuation of future rewards for current sacrifices in terms of consumption foregone has increased.

⁵ And we require $\rho - \hat{P} > 0$, which is a necessary (but not sufficient) condition for the integral in (4) to converge.

⁶ The proportional growth rate of a variable x is denoted by \hat{x} .

It should strike one as odd, although Lucas does not comment on it, that the growth rate of consumption per head rises with the rate of growth of the number of heads. But the underlying reason is very simple, and has to do with the non-rivalrous nature of human capital per person in Lucas' model. Since the total human capital stock is the product of human capital per person and the number of persons,⁷ Lucas' and our specification implies that a larger future population raises the productivity of current human capital accumulation activities, the results of which will be 'shared' in a non-rival way among the individuals of this larger population. Given the specification of (3), a growing population then 'ought' to allocate more resources to human capital accumulation, and consumption per head should grow faster than with a non-growing population.

3. Health extensions to the Lucas model

3.1 Introduction

Health enters the intertemporal decision framework in three different ways. *First*, a fall in the average health level of the population may be expected to cause a fall of the amount of effective labour services that the population can supply. *Second*, the generation of health takes scarce resources that have alternative uses (like the production of output or human capital), while *third*, a good health may be expected to influence utility directly. As we have mentioned above, this direct link consists of the relation between the net growth rate of the population and the endogenously determined level of health activities. We elaborate this below.

⁷ If, instead, the absolute human capital stock as such would accumulate in accordance with (3) rather than average human capital per person, and if human capital per person would be equal to the human capital stock thus accumulated per head of the population, then one can easily verify that the growth rate of the population would drop out of equations (5) and (6).

3.2 The demographical and epidemiological module

We distinguish between two different health states that the population can be in. People are either healthy and then belong to the group of healthy people H, or they are not healthy and belong to the group of non-healthy people S – the latter group also includes the elderly inactive persons. We obviously have for the total population $P = H+S$. The health state of an individual can change either through exogenous causes or through health activities. Such a change is represented by a flow between the two different states, as depicted in Figure 1 below.

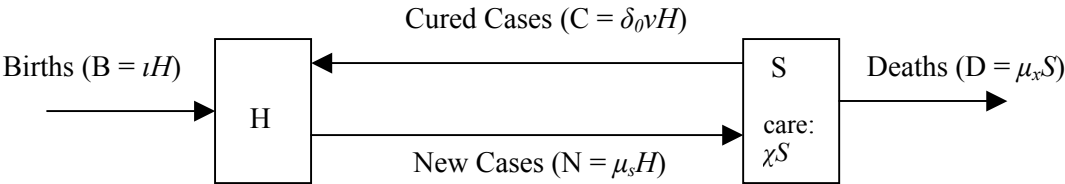


Figure 1 Stock-flow diagram of the population

As Figure 1 shows, we assume that all people are born healthy (B is an inflow in H), and only healthy people reproduce, at a given rate ι . In addition, healthy people don't die (D is an outflow of S). They will do so only if they get sick first. People can move from the healthy state into the non-healthy state at a given morbidity rate μ_s , i.e. a fraction μ_s of all healthy people at a certain moment in time will become ill, or otherwise non-active (cf. the flow N from H to S). A given fraction μ_x of all non-healthy people will die and so leave the population. The number of non-healthy people that switches to the healthy state (cf. the flow C from S to H) is assumed to be proportional to the level of cure activities as given by v.H.

The factor of proportion is a given parameter called δ_0 . The parameters are all positive and smaller than unity, except for δ_0 that can be bigger than 1. Apart from the resources spent on the flow out of S to H, care resources need to be spent on the individuals that stay in S (cf. equation (13) below). The set of assumptions above can be summarised as follows:

$$dS/dt = N - C - D = \mu_S \cdot H - \delta_0 \cdot v \cdot H - \mu_X \cdot S \quad (7)$$

$$dH/dt = B + C - N = \iota \cdot H + \delta_0 \cdot v \cdot H - \mu_S \cdot H \quad (8)$$

Equations (7) and (8) show how the number of healthy and non-healthy people changes over time, and therefore how the population itself changes. Defining $h = H/P$, it follows that $S/P = (1-h)$, and therefore we have from (7) and (8):

$$\dot{P} = \frac{dP/dt}{P} = \frac{dH/dt + dS/dt}{P} = (B - D)/P = \iota \cdot h + (-\mu_X) \cdot (1 - h) \quad (9)$$

Equation (9) defines the net growth rate of the population P as the weighted average of the growth rates of healthy and non-healthy people. One sees immediately that for $\iota > \mu_X$ an increase in h will lead to an increase in population growth, as might be expected.

3.3 Cure activities in the steady state: the aggregate health production function

Our assumption that the number of non-healthy people that is cured is proportional to the level of cure activities as given by $v \cdot H$ is at the base of the aggregate health production

function. The implication for the growth rate of the fraction of healthy people in the population can be found using (8) and (9), which yields:

$$\hat{h} = \dot{H} - \dot{P} = (\iota - \mu_S) + \delta_0 \cdot v - (\iota + \mu_X) \cdot h + \mu_X \quad (10)$$

This equation shows, as one might expect, that a higher effort by the health sector, i.e. a higher value of v , yields a stronger health growth. However, the higher average health level also represents a drag to this growth because of its positive impact on population growth. Following the line taken in van Zon and Muysken (2001) we will concentrate on the steady state properties of the health cure activities, where we expect a constant value of v .

An interesting property of differential equation (10) is that it has a stable equilibrium in the steady state, since $\iota + \mu_X > 0$. As a consequence, for any given positive value of v , the health level h will converge to h^* . The latter can be obtained by setting the growth rate of h as given by (10) equal to zero:

$$\hat{h} = 0 \Rightarrow h^* = \frac{\delta_0}{\iota + \mu_X} \cdot v + 1 - \frac{\mu_S}{\iota + \mu_X} \equiv \zeta_0 \cdot v + \zeta_1 \equiv h(v) \quad (11)$$

where $\zeta_0 = \delta_0 / (\iota + \mu_X)$ and $\zeta_1 = 1 - \mu_S / (\iota + \mu_X)$. Equation (11) defines the steady state health level as a function of v and the parameters defined above. As in van Zon and Muysken (2001), we will use (11) as the aggregate health production function for the steady state. Since it is well-documented that the provision of health services takes place under conditions of decreasing returns (see, for instance, Johansson and Lofgren (1995) and Or (2000)),— we assume that $\zeta_0, \zeta_1 > 0$. This ensures that the average health productivity h/v falls with an

increase in v , and leads to Baumol's law at the macro-level in the case of health services, as we mentioned before.

The requirement $\zeta_1 \neq 0$ implies $\mu_S \pi \iota + \mu_X$, from which follows that the steady state health level in the absence of care activities is still positive, i.e. not everybody is ill. A final restriction follows from the linear nature of the health production function, to ensure that h^* does not exceed 1. One sees immediately that this puts a maximum v^{\max} on the share of the cure activities in total employment, such that $v \pi \iota + \mu_X = \mu_S / \delta_0$ should hold.⁸

It follows from equation (11) for a given value of v , that the steady state health level of the population h^* depends positively on the 'cure productivity' δ_0 , as one might expect. It is also positively related to ι and μ_X . First of all because h increases if ι increases by construction, and secondly $(1-h)$ decreases (hence h increases) if μ_X increases. Finally, h^* decreases if μ_S increases, as it should.

Substituting (11) into (9), we get the steady-state population growth rate:

$$\dot{P} = \delta_0 \cdot v + \iota - \mu_S \equiv \eta_0 \cdot v + \eta_1 \quad (12)$$

where $\eta_0 = \delta_0$, $\eta_1 = \iota - \mu_S$. Comparison with equation (8) shows that this equals the growth rate of the number of healthy people in the population, so that h does indeed remain fixed in the steady state. The stability of (10) ensures that this will be the case in the long run.

Equation (12) shows that as v comes close enough to v^{\max} , i.e. $v^{\max} = \mu_S / \delta_0$, the steady state population P will be growing. Moreover, as one might expect, the growth rate of

⁸ An alternative interpretation of this requirement is that it ensures that the growth of the healthy population does not exceed the birth rate, i.e. $\hat{H} \leq \iota$, as can be seen from equation (8). For, in a steady state growth situation, the growth rate of H can not be permanently higher than the birth rate.

the population depends positively on the ‘cure’ productivity δ_0 , positively on the birth rate and negatively on the rate of morbidity, for given values of v .

3.4 Adding care activities

Unfortunately, not every non-healthy person can be cured. We assume that non-healthy people need and will be cared for. At each moment in time this costs healthy labour resources that are proportional to the number of people that are ill or otherwise inactive, with a factor of proportion χ . Let these resources be a fraction u of the healthy population. Then we have:

$$u \cdot h \cdot P = \chi \cdot (1 - h) \cdot P \Rightarrow u = \chi \cdot (1 - h(v)) / h(v) \quad (13)$$

Since it follows from equation (11) that u depends negatively on v , equation (13) shows that there is a direct trade off between u and v . This represents the notion that a low value of v lowers h and therefore raises the proportion of sick people in the population and hence the need for care activities.

4 Model modifications and formal growth results

Given the assumptions regarding the use of health resources for the cure and care purposes outlined above, the Lucas production structure is modified as follows:

$$Y = A \cdot ((1 - u - v - w) \cdot e \cdot h \cdot P)^\alpha \cdot K^{1-\alpha} \quad (14)$$

$$dK / dt = Y - c \cdot P \quad (15)$$

$$de/dt = \delta_e \cdot e \cdot h \cdot w \quad (16)$$

The difference with equations (1) – (3) is that we recognise that resources that are used for care and cure cannot be used for production. Moreover, only healthy hours (measured in efficiency units) are used in final goods production and in human capital accumulation.

In addition to (14) – (16), we add the macro-health production function given by (11),⁹ and the demand for care activities defined in (13). Finally we also add the endogenous link between cure activities v and the growth rate of the population \hat{P} as given by (12). Hence the full model consists of equations (11) – (16).

A description of all the first order conditions of the model is presented in the Annex to this paper. Here we concentrate on a description of the main results. The solution to the optimum control problem is obtained for the steady state, i.e. the situation where the control variables u , v , w and the real interest rate r are constant.¹⁰ Moreover, we focus on comparative steady states disregarding transitional dynamics.

The solution of the model results in the following equation for the optimum growth rate of consumption per head:

$$\hat{c} = \{\delta_e \cdot (1 - v - u) \cdot h(v) + \eta_0 \cdot v + \eta_1 - \rho\} / \theta \quad (17)$$

This is identical to the Lucas endogenous growth results as given by (5), except for the term $(1 - u - v) \cdot h(v)$ which represents the fraction of healthy working hours available for

⁹ Consistent with the notion of Baumoll's law at the micro level, we do not assume labour augmenting technical progress in the health sector.

¹⁰ If the latter is not the case, then the standard result optimisation result for the rate of growth of consumption per head to be constant under a CES function would not hold.

activities outside the health sector.¹¹ Since Lucas doesn't have a health sector, the same fraction in the Lucas model is 100 percent, in which case (17) is reduced to (5).

Using (11) and (13), equation (17) can be rewritten as a quadratic function of v :

$$\hat{c} = \{-v^2 \cdot \delta_e \cdot \zeta_0 + v \cdot (\eta_0 + \delta_e \cdot (\zeta_0 - \zeta_1 + \zeta_0 \cdot \chi)) + \delta_e \cdot (\zeta_1 - (1 - \zeta_1) \cdot \chi) - \rho\} / \theta \quad (18)$$

Equation (18) describes a parabolic relationship between consumption growth and cure activities v . It increases in v through its impact on population growth and the positive impact of cure activities on productivity. However, the diminishing returns to health production and the increasing absorption of employment necessary for cure and care activities will eventually take over. Hence consumption growth reaches a maximum for $v^* = (\eta_0 + \delta_e \cdot (\zeta_0 - \zeta_1 + \zeta_0 \cdot \chi)) / (2 \cdot \delta_e \cdot \zeta_0)$ and decreases for higher values of v . The parabolic growth equation is depicted in Figure 2 below.

As the Annex shows, the solution of the model also yields a second relation between the optimum growth rate and the volume of cure activities v – which is derived from the dynamic constraint regarding the optimal development over time of the population. Unfortunately, this gives rise to an implicit relation between the growth rate of consumption per head and v , that is strongly non linear and precludes finding an analytical solution.¹² However, the function can be solved numerically and for plausible parameter values it depicts a negative relationship between the growth rate and v , which is also depicted in Figure 2. Part of the explanation for the downward sloping relation is that more cure activities (a higher value for v) increase population growth, cf. equation (12), and hence a lower value of consumption growth is required to achieve the same utility, cf. the intertemporal utility

¹¹ Note that the term $\eta_0 \cdot v + \eta_1$ in equation (17) represents population growth.

¹² See the Annex for further details.

function (4). In general, however, v contributes through many channels to the valuation of population growth. It contributes positively through its impact on the steady state health level of the population, but current output and investment levels are negatively affected by increases in v (because of the reallocation of resources this entails). Hence it depends on the specific parameter values whether a negative relative steep relation with a unique equilibrium solution will arise. Since we found that this is the case for plausible parameter values, we will use that curve to illustrate the working of our model.

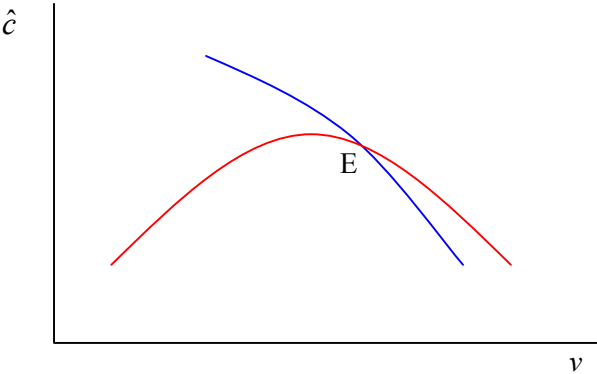


Figure 2. The growth and health trade-offs

In van Zon and Muysken (2001) we could provide convincing arguments for the location of the equilibrium point E to the right of the top of the parabolic growth curve.¹³ Lack of analytical tractability precludes us from coming up with any non-numerical arguments here. The outcomes we present below, however, resemble the general situation depicted in Figure 2, as well as in van Zon and Muysken (2001).

¹³ In this model E can be anywhere, which is not necessarily a bad thing, certainly for economies that have a point of intersection to the left of the top. If these economies could somehow shift the downward sloping curve upwards, then they could experience a double dividend, i.e. higher growth and higher health.

5. A numerical analysis of the link between health and growth

Although it is not possible to provide a full analytical solution of the model, with some reasonable a priori parameter values, one can obtain a graphical impression of both relations between the growth rate of consumption per head and the level of cure activities, as Figure 2 illustrates. The underlying parameter values have either a priori reasonable values, or they are calibrated such that the growth outcomes and the general behaviour of this growth system seems reasonable. Nonetheless, there is an arbitrary element in the presentation of the results below, since the non-linearity of the system does allow for multiple equilibria and a host of other computational problems. We leave this for future scrutiny, however. Suffice it to say here that the model does generate qualitatively the same results as in van Zon and Muysken (2001), although we now have an endogenously evolving population. The parameter values are presented in Table 2.

'Lucas' Parameter	Value	Health Parameter	Value	Implied Structural Parameter	Value
θ	0.5000	χ	0.1000		
α	0.7500	η_0	0.0100	δ_0	0.0100
ρ	0.0500	η_1	0.0050	μ_S	0.0089
δ_e	0.1000	ζ_0	0.4500	μ_X	0.0083
		ζ_1	0.6000	ι	0.0139

Table 2. Parameter values.

The first column of Table 2 shows the standard parameters of the Lucas model, of which the numerical values are roughly similar to those used in van Zon and Muysken (2001).¹⁴ Since the health sector is modelled somewhat differently, we chose new parameter values for the health parameters in the third column, which are presented in the fourth column of Table 2.

The values for the health parameters from column 3 in Table 2 first were constrained to obtain reasonable results for the structural parameters shown in columns 5 and 6 of the table. The steady state share of healthy persons $h = 0.65$ implies a birth rate relative to the entire population close to 0.9 percent, and a death rate that is somewhat low at 0.3 percent. These numbers result in a population growth rate equal to 0.63 percent. These outcomes have all the right order of magnitude.¹⁵ Next the health parameters were calibrated in order to obtain reasonable values for the growth rate of consumption per head of about 2 percent, and for the allocation of labour time over its various uses, i.e. care and cure between 10 and 15 percent, whereas u should be twice as small as v , human capital accumulation of about 30 percent and the remainder on final output production. The figure on human capital accumulation is in line with Lucas' (1988, p.26-27) finding that 28 % of total working time is spent on human capital formation. The care and cure figures are based on the situation in the Netherlands, where the share of health expenditures in GDP has now risen above 10 percent, due to the present lack of economic growth and the ever increasing costs of health care.¹⁶ In addition to this, the distribution of total health costs between care and cure activities is roughly equal to 35% and 65%, respectively.¹⁷ The precise steady state outcomes are listed in the second column of Table 3 below, labelled SSV. Inspection of Table 3 shows that the

¹⁴ There we used $\theta = 0.5$, $\alpha = 0.65$, $\rho = 0.075$, and $\delta_e = 0.114$ resulting in a growth rate of 0.026. The somewhat different values for θ and ρ follow from the interaction with the different health structure of the present model in the calibration process.

¹⁵ The figure for h seems too low, but we will later on reinterpret this number as the share of active people in the population, rather than the share of healthy people in the population.

¹⁶ The cost share is admittedly a very rough indicator of the labour share, but Baumol's law suggests that the 10 percent would be a lower-estimate for the volume share of labour.

¹⁷ See www.rivm.nl/kostenvanziekten for information about the costs of illness in the Netherlands in 1999. The distribution of total costs over different forms of cure and care, suggest that care activities take of the order of 36% of total costs in 1999.

fraction of time spent on health activities, in total about 18 percent, is a bit high, as is the time spent on human capital accumulation – roughly a third of one’s life. Nonetheless, none of the variables seem to be wildly off a priori – which fits our illustrative purpose. Moreover, the parameter restrictions on the model which we formulated in the previous section are all satisfied.¹⁸

<i>Var</i>	<i>SSV</i>	θ	α	ρ	δ_e	χ	μ_s	μ_x	ι	δ_0
\hat{c}	0.020	-1.27	0.08	-4.97	5.61	-0.40	-5.18	1.62	4.09	0.08
u	0.052	-0.60	0.15	-0.12	0.54	0.88	0.79	0.10	0.21	-1.49
v	0.126	2.45	-0.60	0.49	-2.10	0.42	3.98	-2.69	-4.65	4.76
w	0.311	-1.45	0.14	-4.99	4.45	-0.43	-5.05	1.66	4.19	-0.44
1-u-v-w	0.516	0.34	0.05	2.93	-2.25	0.07	2.01	-0.36	-1.42	-0.76
h	0.657	0.21	-0.05	0.04	-0.18	0.04	-0.26	-0.04	-0.07	0.54
\hat{p}	0.006	0.49	-0.12	0.10	-0.42	0.09	-0.62	-0.54	1.28	1.26

Table 3 Parameter elasticities of health and growth performance

The remaining entries in Table 3 are the elasticities of the steady state equilibrium values for the system variables in the first column with respect to changes in the system parameters. We calculated these elasticities by changing the parameter under consideration by 10 percent, after which we were able to calculate the relative change in the system variables given in the first column. Such a parameter change results in a shift of both curves in Figure

¹⁸ That is, μ_s is smaller than $\iota + \mu_x$, and v is smaller than $v^* = \delta_0/\mu_s$.

2.¹⁹ The shift in the parabolic curve is relatively easy to retrace given equation (18), but that in the downward sloping curve can only be determined numerically.

From the table we can conclude that it reproduces the results found in van Zon and Muysken (2001); raising θ and ρ lowers growth and tips the allocation balance in favour of activities that stimulate current utility rather than future utility, i.e. final output production and health production (because of its positive impact on current productivity levels). Parameter changes that affect the productivity of the health sector directly through δ_0 and χ , or in a broad sense through the compound parameters $\eta_0, \eta_1, \xi_0, \varsigma_1$, also work in the same way as in van Zon and Muysken (2001). Except perhaps for changes in the rate of mortality, the results obtained for the other structural parameters are plausible a priori. We will further elaborate the results on mortality in the next section where we analyse the decrease in mortality that drives the ageing of the population in Western economies.

From the parameter elasticities in Table 3 we make five observations which illuminate the working of our model. First, as might be expected from our discussion in equation (13), we observe a trade-off between cure and care activities. Our results show that, as a rule, u and v move in opposite directions, indicating that there exists a structural trade-off between both. In fact, that trade-off is underlined by the exception to the rule provided by the results regarding the sensitivity to changes in χ . A rise in the latter parameter implies that the care intensity of non-healthy people rises, which increases the marginal benefits from cure activities. This positive second order effect on u does not totally remove the increased need for care due to a rise in χ , however. The observation that for an increase in the morbidity rate we observe a positive change in both u and v can be explained in a similar way, as we elaborate in the next section.

¹⁹ We also calculated the parameter elasticities for a drop in the parameters by 10 percent. These gave qualitatively identical results, and numerically nearly identical results, that are therefore not presented here.

Secondly, there is a very robust trade off between cure and human capital accumulation. It is these activities that have relatively strong intertemporal effects through their impact on the growth rate of the population, and on the growth rate of human capital per person.²⁰ Interestingly, there is a positive correlation between changes in v and $1-u-v-w$, suggesting that a change in v is primarily countered by an opposite change in u and w . Apparently, the rise in v raises the marginal product of labour in final output production more than proportionally (i.e. both through the rise in $h(v)$ and through the fall in $1-u-v-w$, ceteris paribus).

A third observation is that an increase in productivity in education or cure activities does not lead to an increased share of labour available for production, i.e. substitution effects outweigh the corresponding income-effects. Note that this finding is consistent with the observed positive correlation between productivity increases in the health sector and the increased share of health expenditures in GDP (Glied, 2003).

Fourth, it is interesting to note that a rise in the productivity of cure activities, even though the equilibrium is on the downward sloping part of the parabolic curve, does give rise to a double dividend, since both the steady state health level of health, and the rate of growth of consumption per head are positively affected, whereas in all other cases a negative correlation exists between the impact on growth and the impact on the steady state health level of some parameter change. So, health seems to be a substitute for growth.

Finally, a decrease in the relevance of the future in deciding what to do now (either a rise in θ or in ρ), causes a rise in the steady state growth rate of the population, through a reallocation of human capital accumulation time to cure and production activities. It is equally interesting to notice that cure activities rise much faster than the allocation of labour time to

²⁰ This is in line with what we found in van Zon and Muysken (2001). Our finding that v and u are negatively correlated, is in contrast with micro economic analysis, which suggests a positive correlation – cf. Fuchs (1982) and Bloom and Canning (2000). The latter point out that first of all an increased health induces a higher productivity of learning (a modern variant of “mens sana in corpore sano”) and secondly induces more investment in human capital through the potential of higher returns.

final output production, because cure activities positively influence the availability of labour time for all activities. However, there is a counter effect resulting from the observation that a decrease in the discount rate will result in higher growth and less resources used up in cure activities. Since such a decrease is consistent with increased longevity (Becker and Mulligan, 1997), this might point to a positive feedback between longevity and economic growth.

In conclusion we may state that the comparative steady state analysis we have performed here seems to generate a priori plausible results, that are in line with the results by van Zon and Muysken (2001), even though the steady state population growth rate itself now depends on care and cure activities. The question we will address in the next section is how we might interpret the changes observed in Western economies, i.e. the ageing of the population, into corresponding changes in terms of our model, and see what the comparative steady state growth implications will be.

6. Growth impacts of demographical changes

The reason why we want to have a look at the burden of an ageing population from an endogenous growth perspective is that these effects can be significant in principle, but are routinely overlooked when using the standard apparatus for such exercises. Jacobzone et al. (2000), for instance, use a simple extrapolation scheme to obtain an estimate of the share of health spending in GDP, by multiplying the current share with the relative increase over time in the dependency ratio as compared to its current value. The numerator of the dependency ratio is a direct indicator of the projected increase in health demand, whereas the denominator is a direct indicator of the projected increase in the productive base of the economy. However, such a mechanistic extrapolation scheme is incomplete and overly optimistic, certainly in the long run, since a rise in the dependency ratio through an increase in longevity, and the

allocative adjustments this entails, may act as a break on economic growth, as our sensitivity analysis in section 5 has indicated.

Because care activities seem to be especially insensitive to labour augmenting technical change, Baumol's law holds strongly here. The need to supply care to the older generation when they need it will therefore inevitably lead to a changing distribution of labour time in favour of health care, and so add a significant growth dimension to the level problems caused by an ageing population. That this growth dimension may indeed be significant, follows directly from the relative sensitivity of the growth responses to changes in demographical and epidemiological parameters on the one hand (see Table 3), as well as the significance per se of the demographic changes expected for the future²¹ that ultimately drive changes in the dependency ratio in the long term. At this stage, however, we can only illustrate the existence of a case in favour of a general equilibrium growth perspective in medical spending decision making, rather than providing a complete numerical defence, although, as stated above, the circumstantial evidence seems to be strong.

So far, we have described our model in terms of a population that can be in two different health states, i.e. healthy and non-healthy. However, from an economic perspective the distinguishing feature between people in these two states is that they differ in terms of production and consumption activities. Non-healthy persons do not produce anything, whereas they do consume final output and health services. Healthy persons on the other hand only consume final output. In addition they produce both output and health services. Furthermore, they accumulate human capital. It does not take a giant leap of the imagination to see that the consequences of a permanent change in the composition of the population in terms of active and inactive people in favour of the inactive due to an ageing of the population will resemble those of a change in the composition of the population in our model in favour of

²¹ The dependency ratio's from Table 1 are expected to double more or less between 2000 and 2030, certainly for the Netherlands (see OECD (2003)).

non-healthy people, *ceteris paribus*. We analyse these consequences below by mimicking a more or less autonomous drift in the composition of the population in favour of inactivity by manipulating the demographic and epidemiological parameters of our model. We also investigate what our model has to say about the growth effects of policy actions aimed at reducing the dependency ratio.²²

Growth effects of an ageing population

In the context of our model, an ageing population can be mimicked by a rise of the propensity to stay in S once one is in S. The latter can be linked directly to a fall in the rate of mortality μ_x . Since that leads to a rise in the average age of the population, and since older people generally require more care than younger people, this would also imply an increase in χ . At the same time one would also have to lower the productivity δ_0 of curing activities, since one cannot normally be cured from inactivity due to old-age.

We are now able to infer, albeit in a fairly impressionistic way, what an ageing population may mean for growth. The growth effects of changes in the parameters μ_x , χ and δ_0 all work in the same direction. A fall in mortality reduces the growth rate, and so does an increase in care intensity and a decrease in cure productivity. Hence growth of the economy will be affected in an unambiguously negative way. The effects on the steady state health level are less clear cut, although it seems likely that the limited positive effects of a decrease in mortality and an increase in care intensity are more than balanced by the negative effect of a drop in cure productivity δ_0 . The negative growth effects are mirrored by a net decrease in w , but it is hard to say how the reallocation of resources will affect the distribution among v, u

²² At present, the Dutch government considers a rise in the retirement age as part of the solution to decreasing the financial burden of an ageing population.

and 1-u-v-w. It is equally hard therefore to conclude what will happen to the health sector as a whole. It is clear, though, that the balance of activities will shift in favour of cure and of current output production, thus diminishing the rate of growth due to the ageing of the population. This finding is in line with the one for a constant population by van Zon and Muysken (2001) who indicated that an ageing population would lead to a fall in the overall productivity of the health sector. In the current model too, a fall in the productivity of health activities either through a fall in δ_0 or an increase in χ , leads to a definite fall in growth performance.

Growth effects of early retirement

The effects of a decrease in the dependency ratio by raising the (early) retirement age could be captured by lowering the rate of morbidity μ_s to mimic a decreased flow from the active state to the inactive state, increasing χ to mimic the fact that early retired people require less care on average than older retired people, and to increase δ_0 in order to mimic the rise in the responsiveness of the inactive population to cure activities.²³ A decrease in the rate of morbidity has a very large positive effect on the rate of growth of consumption per head (see Table 3). The negative impact on growth of an increase in care intensity is of a relatively limited size, however, whereas the rise in δ_0 with a rise in the retirement age has positive but equally limited growth effects. The positive growth effect of the increase in morbidity that mimics the postponement of early retirement is thus likely to dominate. As with the ageing of the population, the net effect on health activities is unclear. Nonetheless, increasing the retirement age almost certainly helps to counter the negative growth effects of an ageing population. In fact, if the difference in the sensitivity to changes in the rate of mortality and in

²³ A higher fraction of inactivity is now due to ill-health.

the rate of morbidity is as significant as shown in Table 3, then we may even expect that discouraging early retirement would stimulate growth more than an ageing of the population would depress it. From an economic perspective this also makes sense, since early retirement moves active people into inactivity, thus reducing the productive human capital base of the economy, while an ageing population only raises the ratio of inactive versus active people, without actually reducing the absolute number of inactive people.

7. Concluding remarks

Having a good health care system and being cared for in a decent manner when old comes at a cost in terms of current consumption and growth possibilities foregone: intergenerational decency comes at a price. Acknowledging that is but part of the solution to the problem of financing health activities that generally seem to be perceived as a continuously growing burden on Western economies due to an ageing population, that increases the demand for care and cure per head, but also the number of heads, *ceteris paribus*. It is the very nature of these care and cure activities that call for a growing share of the health sector in GDP, due to Baumol's law.

By incorporating Baumol's law in our model, we have shown that steady state growth situations are still possible, even though they are influenced by population dynamics. Different steady state growth situations can arise for different structural parameter combinations. Therefore policy actions impacting directly on those parameters (for instance through the productivity of cure and care activities) may have an equally direct impact on health production, and hence also on growth performance. To some extent we can choose between different health and growth futures.

In addition to these health productivity promotion actions that would increase the effective availability of scarce labour resources, other types of policy actions may prove to be at least as effective. Our results suggest that an increase in the retirement age may be effective in promoting growth and sustaining high levels of health.

Of course, in this paper we have concentrated on the promotion of health as a prerequisite for growth, but the ultimate source of growth in this model is still technical change. Nonetheless, our model has been built on the basic principle that a good health is instrumental in realising the productivity potentials provided by ever improving production technologies. From the sensitivity analyses above, we have seen that growth performance is highly sensitive to changes in the productivity of human capital accumulation activities (i.e. δ_e), that are at least as important for growth as changes in morbidity rates, mortality rates, and so on. Indeed, the results obtained from our model suggest that it may be wise to include the growth effects of health spending decision making in economic policy analysis, since an exclusive focus on cutting current health costs rather than focussing on the intertemporal effects of health activities too, may have severe negative effects for long run growth performance, simply because having a good health is necessary for every individual to realise his or her productive potential. And although underachievement with respect to the provision of health services has a direct negative effect on welfare, it is perhaps even more important to consider that it negatively affects the productive base of the economy and the economy's potential for growth.

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Technical Annex

The Hamiltonian of the revised Lucas model can be written as:

$$H = e^{-\rho t} P c^{1-\theta} / (1-\theta) + \lambda_K (A((1-u-v-w).e(\zeta_0.v + \zeta_1).P)^\alpha K^{1-\alpha} - c.P) + \lambda_e \cdot w \cdot \delta_e \cdot (\zeta_0.v + \zeta_1).e + \lambda_P \cdot P \cdot (\eta_0.v + \eta_1) \quad (\text{A.1})$$

where c , u , v and w are the control variables, K , e and P are the state variables, and where we have used equations (11),(12), (14)-(16). $\lambda_K, \lambda_e, \lambda_P$ are the co-state variables of K , e and P . They measure the marginal value (in terms of integral utility) of an additional unit of the respective stock at a certain time. By substituting (13) into (A.1), u can be dropped as a direct control variable. The first order conditions with respect to the remaining control variables are:

$$\frac{\partial H}{\partial c} = e^{-\rho t} c^{-\theta} \cdot P - \lambda_K \cdot P = 0 \quad (\text{A.2})$$

$$\frac{\partial H}{\partial v} = \lambda_K \partial Y / \partial v + \lambda_e \partial (de / dt) / \partial v + \lambda_P \cdot \partial (dP / dt) / \partial v = 0 \quad (\text{A.3})$$

$$\frac{\partial H}{\partial w} = \lambda_K \partial Y / \partial w + \lambda_e \partial (de / dt) / \partial w = 0 \quad (\text{A.4})$$

Equation (A.2) gives rise to the familiar growth $\hat{c} = (-\hat{\lambda}_K - \rho) / \theta$. Equations (A.3) and (A.4) state that on an optimum path, total utility at some point in time cannot be improved upon by shifting resources between their alternative uses. The dynamic constraints state that on an optimum path total utility can not be improved upon by shifting resources that can be accumulated (the state variables) over time, i.e. by changing the rate of investment, human capital accumulation and the rate of growth of the population. This will be the case if the

valuation of an additional unit of a stock falls by exactly the same amount as the direct contribution of that additional unit to current utility (this includes the impact of that additional unit on future stocks and the contribution of future utility to total utility as captured by the co-state variables). We therefore have as dynamic constraints:

$$d\lambda_x / dt + \partial H / \partial X = 0 \quad \forall X = K, e, P \quad (\text{A.5})$$

Equations (A.5) give rise to a set of differential equations in the co-state variables. Since (A.2)-(A.4) must hold for all t, the time derivatives of (A.2)-(A.4) must also be equal to zero. These results can be substituted into the system given by (A.5). Using equation (A.5) for $X=K, e$ in combination with the constraints (A.2)-(A.4) as well as (11), (12), (14)-(16), and assuming a fixed allocation of labour time, we arrive at equation (18). Using (A.5) for $X=P$ and using the same set of constraints as before, we end up with an implicit relation between the growth rate of per capita consumption and v . The latter relation is obtained by using some intermediate results from the previous step, notably the results $\hat{c} = \hat{e}$, $\hat{c} = (r - \rho) / \theta$, $\hat{K} = \hat{e} + \hat{P}$ and $\hat{\lambda}_p = \hat{e} - r$. In addition to that, we use the investment constraint to solve for consumption $c = (K/P) \cdot (r / (1 - \alpha) + \hat{K})$. Likewise, we use the production function to relate K/P to the real rate of interest, and given the intermediate results above, to \hat{e} again. The resulting link between c on the one hand and \hat{e} and v on the other is used to substitute for c . In addition, w is substituted for by using (16), again giving rise to an expression containing both \hat{e} and v . All these manipulations lead to a strongly non-linear implicit relation between \hat{e} and v , which location and shape in the \hat{e} , v -plane depends very much on the particular parameter values chosen/observed.

Further technical details are available on request in the form of a Mathematica notebook.