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On The Variance of Market Innovation with the Number of Firms

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Abstract

This paper models a dynamic innovation process to examine the relationship between levels of R&D and market structure. In contrast to most of the literature the model includes positive feedbacks within the R&D process of a firm, wherein one output of R&D is knowledge, and accumulating knowledge in the present makes future R&D less costly. This creates a feedback by which market structures can affect steady state levels of R&D. We find that in general while an increase in the number of firms in an industry reduces R&D per firm this effect is sub-linear so industry R&D increases. The model also endogenizes the number of firms, using a zero profit condition. Finally, welfare effects are discussed.

JEL Classification: L00, L01, L02, O3.

Keywords: Innovation, R&D, Market structure, firm size.

1 Introduction

The relationship between firm size or market structure and the level of R&D has been debated by economists since the issue was raised by Schumpeter early in the twentieth century. There has been concern not only with the effect market structure has on the quantity of R&D, but also whether different market structures generate sub-optimal levels. Theoretical work has generated contradictory results—there is no consensus about whether the amount of R&D at the industry level is positively or negatively related to firm size.[1] Empirical investigations have been similarly inconclusive.[2] Surveys of the empirical literature often summarize with statements like "The main characteristic of the literature on the innovation-market structure hypothesis is its inconclusiveness." (Symeonidis, 1996, p. 16)

Sah and Stiglitz (1987) present a strong argument that there should be no systematic relationship between firm size and R&D output, which would explain the inconclusiveness of the empirical literature. This result has been widely cited, largely due to the elegance and simplicity of the argument. In its barest essentials, the argument runs as follows: Think of R&D in terms of projects. There may be many projects which provide potential means to producing a new product, and firms may engage in more than one project. If a project is profitable (or has positive expected profits), then some firm will do it. Thus regardless of the number of firms in an

[1]See for example, Dasgupta and Stiglitz (1980), Dixit (1988) or Loury (1979). Traditional arguments that large firms or concentrated industries will generate more R&D involve fixed costs of R&D; economies of scale and scope in production of innovations; risk spreading among projects; access to external finance and ease of self-finance. On the other hand small firms or un-concentrated industries benefit because of decreasing returns to scale in production of innovations; lack of bureaucratization of the R&D process; strong competitive pressures from the market demanding innovation. More recently competitive effects have been introduced explicitly, for example through the "efficiency effect" whereby monopoly can internalize the externalities of creative destruction (Gilbert and Newbery, 1982). But the ambiguity remains. Greenstein and Ramey (1998), for example, argue that "competition and protected monopoly provide *identical* incentives for innovation [but that] ... threatened monopoly provides *strictly greater* incentives" (p. 286).

[2]See studies by Mansfield (1981) or Geroski (1989) for evidence that industry concentration or firm size has a negative impact on R&D. See Baldwin and Scott (1987), or Lieberman (1987) for evidence of a positive relationship. Bound et.al. (1984); Cohen, Levin and Mowery (1987); Jensen (1987); and Scott (1984) find no relation. See also Kamien and Schwarz (1982) or Symeonidis (1996) for surveys.

industry, the same number of projects will be undertaken.[3]

This is a powerful argument, and certainly a valid one. It may not be sound. The argument contains an implicit assumption that the number of firms in the industry does not affect the profitability of projects. This is a debatable claim. On the output side, the presence of competition effects would contradict this claim (see e.g. Gilbert and Newbery, 1982; or Greenstein and Ramey, 1998.) The claim is questionable even ignoring these effects and concentrating solely on the production of innovations through R&D.

The profitability of a potential R&D project is a function of many things, some of them inherent in the project itself, but one which is not, namely the knowledge base of the firm considering doing it. Clearly, in general, the bigger the knowledge base of a firm, the easier it will be for it to perform any particular research project. Knowledge arises from many sources, but one is the R&D process itself. R&D produces new products and processes when successful, but it does more than this. Even when it is not successful, either on technical or market criteria, R&D produces knowledge.[4] Knowledge accumulated by performing R&D changes the knowledge base of the firm, and may have an effect on the profitability of engaging in further research. This is a form of dynamic increasing returns in which more R&D today generates higher R&D productivity tomorrow. While clearly an important force in any ongoing R&D process, this effect is largely missing from the literature.[5]

This observation is of particular interest when considering industries in which R&D is a continual process. It is seldom the case that a firm will do a single R&D project, with no plans to do others in the future. When research is ongoing and when knowledge accumulates with that research, in order to understand R&D and,

[3] This is a significant simplification of their argument.

[4] See for example Cohen and Levinthal (1989), or Foray and Mowery (1990) for discussions of this feature of R&D.

[5] Peretto (1996), in a general equilibrium growth model, includes static increasing returns to scale in R&D, and dynamic increasing returns in goods production. Yi (1999), in a model of firm incentives to innovate, also includes dynamic increasing returns, here feeding back from output to the quantity of R&D. In neither model, though, are there any effects by which R&D feeds back onto its own productivity.

more specifically, the relation between the quantity of R&D and industry structure, a dynamic treatment is necessary.[6]

This paper presents a model of the R&D process, in which firms undertake R&D projects knowing that there is a positive probability that any project will fail. The model can be seen as repeated patent races, so that rents are allocated *ex post*: winner-take-all in each race. R&D raises (expected) profits, but also produces knowledge that is useful in performing future research. When this link between present and future R&D is acknowledged, a relationship between the number of firms in an industry and total industry R&D emerges. The direction of the relationship depends on various parameter values and on the details of the R&D process but the intuition is straight-forward. R&D *possibilities* are treated as exogenous, and firms choose some number of projects from among the possible ones. The optimal number of projects for a representative firm is affected by the number of firms in the industry. Thus changing the number of firms in an industry changes the amount of R&D a firm performs this period (leaving constant industry R&D in this short run), and so changes the amount of knowledge it has next period. The amount of knowledge is also a factor in the R&D decision, though, which implies that the dynamic path of R&D not only for the firm but also in the industry will be affected by the path of the knowledge stock, which is in turn affected by the number of firms. Clearly, if this sort of effect exists the invariance result of Sah and Stiglitz (1987) is not applicable.

The paper develops a model of the R&D process in which firms undertake risky R&D projects. The process is modelled as a repeated, winner-take-all patent race. Both myopic and forward-looking firms are considered. A numerical example of the model is examined, and its welfare properties are discussed.

[6]Acknowledging the dual output of R&D activity provides a way of attacking a problem in the literature to which Dixit (1988) refers (p.326), namely the difficulty of incorporating partial success in the R&D process. The introduction of knowledge and its accumulation allows that R&D can be partially successful in the sense that it fails to produce an innovation (e.g. not winning the patent race) but produces economically valuable knowledge or human capital nonetheless.

2 A Repeated Patent Race

Consider an industry in which firms are continually doing R&D. Projects are risky, though, so the probability of success of any project is less than one. Firms are Bertrand competitors, so positive rents from R&D only accrue if a firm is the only firm to succeed. A successful innovation creates a temporary monopoly, but the monopoly is destroyed when the next innovation occurs.[7] The model can be seen as a repeated patent race, and as such, provides a dynamic treatment of the issues that Sah and Stiglitz (1987) and others analyze in a static framework.

The industry has n identical firms. In each period, t, a continuum of research projects, labelled [0,1], are possible. Firm j does m_j projects. Each of its projects is successful with equal probability $p(k_j)$ where k_j is the knowledge stock of firm j; $p(\cdot)$ is concave so $0 < p' < \infty$, $p'' < 0$; further, $p(0) \geq 0$.[8] Within a firm, knowledge is a public good so that an addition to the knowledge stock of a firm increases the probability of success on every project. The costs of undertaking a project, c, are identical for every project. Total industry R&D is defined to be M_t in period t. For individual firms, each project undertaken adds to the future knowledge stock of the firm, so knowledge evolves as $k_{t+1} = f(k_t, m_t)$, and we assume that $f(\cdot)$ is concave in both arguments. Knowledge depreciates, so $f(k, 0) \leq k$; and finally, $f(0, 0) = 0$. Both knowledge, k, and the number of projects, m, are non-negative, and the probability of success, $p \in [0, 1)$. Incentives to do R&D are affected by supply considerations, through the probability of success, and demand considerations, through the value of successful innovations.[9]

[7] This structure for the creation of rents is a typical "creative destruction" dynamic. See eg. Aghion and Howitt (1992).

[8] $p(0) \geq 0$ allows the possibility of starting the R&D process with zero knowledge.

[9] Sutton (1991), examining the relation between R&D and market structure, and in particular whether there exist bounds on the relationship, focusses on the same two (industry-specific) properties as central in the explanation. Sutton discusses the elasticity of the cost of R&D function; and the degree of product differentiation. In the current model the relation between knowledge and probability of success, $p = f(\cdot)$ is similar in spirit to Sutton's first parameter; his second clearly affects profitability of innovation: π in the current model. Yi (1999) also emphasizes the demand side, through the elasticity of the inverse demand function. Empirically, Cohen et al. (1987) found that industry-specific appropriability of innovations (which clearly affect the extent

Firms are Bertrand competitors so the payoff to a firm is

$$P = \begin{cases} \pi, & \text{if at least one of its projects is successful and no other firms' are,} \\ 0, & \text{if any other firms' projects are successful} \end{cases}$$

Assuming that the probability of success for different projects is independent,[10] the probability that at least one of firm j's projects is successful is $q_j(k_j, m_j) = 1 - (1 - p(k_j))^{m_j}$. The probability that no other firm has a successful project is $h_j(\mathbf{k}, \mathbf{m}) = \prod_{i \neq j}(1 - p(k_i))^{m_i}$ where $\mathbf{k}$ and $\mathbf{m}$ are vectors of knowledge endowments of, and numbers of projects undertaken by, firms other than firm j. Thus the expected one-period profit of firm j is $EP = \pi h_j q_j - c m_j$.

2.1 Myopic Firms

Consider first myopic firms, which maximize their one-period profits with respect to the number of projects undertaken, treating h as fixed by the actions of the other firms. For an arbitrary firm, dropping the j subscript, the problem is written as

$$\max_{\{m\}} EP = \pi h q - cm, \tag{1}$$

and the first order condition,

$$\frac{\partial EP}{\partial m} = \pi h \frac{\partial q}{\partial m} - c = 0 \tag{2}$$

can be written as

$$c/\pi = \prod_{i \neq j}(1 - p(k_i))^{m_i} \times [-\ln(1 - p_j)(1 - p_j)^{m_j}]. \tag{3}$$

The number of firms, n, enters the first order condition through the product term, and thus can affect firm levels of R&D. It is the case, though, that it does not

of the temporary monopoly power an innovation provides, and so in the current model are included in π), and technological opportunity (included here as part of $p = f(\cdot)$) explained much of the differences in firm R&D levels.

[10] This assumption is not critical, but greatly simplifies the mathematics.

affect industry R&D levels.[11] This is made explicit by examination of the symmetric equilibrium.

In a symmetric equilibrium, $m_i = m_j$ and $k_i = k_j \forall i, j$, implying that the function $h(\cdot)$ reduces to $h(k, m_j) = (1 - p(k))^{M - m_j}$, where M is the total number of projects undertaken by the industry. Dropping the j subscript and substituting, the first order condition becomes

$$c/\pi = (1-p)^{M-m}[-\ln(1-p)(1-p)^m], \tag{4}$$

or

$$c/\pi = -\ln(1-p)(1-p)^M. \tag{5}$$

Solving for M,

$$M = \ln\left(\frac{-c}{\pi \ln(1-p)}\right)\left(\frac{1}{\ln(1-p)}\right), \tag{6}$$

which again is not a function of n.[12]

Differentiating the first order condition with respect to p and M gives

$$\frac{\partial M}{\partial p} = \left[\frac{1}{(\ln(1-p))^2}\frac{1}{1-p}\right] \times \left[1 + \ln\left(\frac{-c}{\pi \ln(1-p)}\right)\right], \tag{7}$$

which is ambiguous in sign. We can see, though, that

$$\frac{\partial M}{\partial p} \gtreqless 0 \text{ as } p \lesseqgtr 1 - \exp\{\frac{-ce}{\pi}\}. \tag{8}$$

so in general if p is small, $\frac{\partial M}{\partial p} > 0$ and if p is large $\frac{\partial M}{\partial p} < 0$. That $\partial M / \partial p$ could be negative seems an odd result, since if the probability of success increases, the firm is more likely to have a successful project. The confounding factor is that the probability of all other firms failing to succeed falls. Since the probability of all other

[11] $\frac{\partial^2 EP}{\partial m_j^2} = -\pi h[(\ln(1-p_j)^2(1-p_j)^{m_j}] < 0$, implying that the optimum is a maximum.

[12] Note here that M is positive only if $p > 1 - e^{-c/\pi}$. If $c/\pi = 0.01$, $p > 0.095$ is the necessary condition for $M > 0$.

firms failing decreases faster than the probability of success for one firm increases, the expected profit of a marginal project declines.

The first order condition defined above generates the *total* industry R&D, M, as a function of the success probability p, which is determined by the representative firm's knowledge level, k. The evolution of M, then, will be determined by the evolution of p, and so by the evolution of k. In a symmetric equilibrium each firm will do M/n projects, and thus a larger number of firms implies fewer projects per firm.

Now consider the next period for the representative firm: $k_{t+1} = f(k_t,\ M(p, k_t)/n)$. $\partial k_{t+1}/\partial n = f_2 \times -M/n^2$, which is negative. Thus the more firms there are in an industry, for a given level of knowledge in period t, the lower the knowledge level in period $t+1$. The independence between total industry R&D and number of firms is broken. Within a single period (or in a one-period model) only the total quantity of R&D is determined, and this industry level is determined by the amount of knowledge in the industry. In an inter-temporal setting, though, the number of firms determines the evolution of the knowledge base, which in turn determines the evolution of industry R&D.

The analysis can be pursued by an examination of the steady state of the system.

Proposition 1: A steady state, defined by $k = f(k, M(p(k))/n)$ exists.

Proof: Consider an arbitrary m. $f(\cdot)$ is concave in k, and $f(0, m) \geq 0$, thus for an arbitrary m a fixed point, $k^* = f(k^*, m)$ exists. Concavity implies that at the fixed point, $f_1(k, m) < 1$. Now differentiate $f(k, m) - k = 0$, to get $dm/dk = -(f_1 - 1)/f_2 > 0$. Thus in m, k space $k = f(k, m)$ has positive slope everywhere, and since $f(0, 0) = 0$, it passes through the origin. Fixing n, and noting that $m = M/n$, from equation 8 and $\partial p/\partial k > 0$, the optimal $m = m^*(k)$ is concave in k, with negative slope for large k. Therefore, if $m^*(0) > 0$, $m^*(k)$ and $f(\cdot) - k = 0$ intersect at an interior fixed point. Otherwise, since $m^*(\cdot)$ is bounded below by 0, there is a fixed point at $m^* = 0$, $k = 0$; and possibly also at an interior point.

Stability properties of the steady states are easy to describe. In general if one or more interior steady states exist, an interior steady state is stable. If there are two interior steady states, the larger value of k is stable. The stability of the origin depends on the behaviour of $m^*(k)$ for $k > 0$ but small. If $m^*(k) > 0$ for $k = 0$, then the origin is unstable. However if there is a $k_1 > 0$ such that $m^*(k_1) = 0$, then the origin is again stable. The former seems more likely since if $m^*(0) = 0$ then without some exogenous shock, no industry could start.

Finally, notice again that n appears in the expression for k_{t+1} implying that in general the value of the fixed point of k is dependent on the number of firms in the industry, and so the stable number of projects M, is dependent on the number of firms. As n increases, the curve $k = f(k, M/n)$ rotates clockwise around the origin. (See Figure 1.) Whether this increases or decreases the steady state value of M depends on functional forms. Define a value $\tilde{k}$ by $p(\tilde{k}) = 1 - \exp\{-ce/\pi\}$. (Shown in Figure 1.) Differentiating $M = mn$, $dM/dn = n \times dm/dn + m$. dM/dn will only be negative if dm/dn is negative, which can only occur if the steady state k is larger than $\tilde{k}$. This will occur if either $k = f(k, m)$ is relatively flat, or if $\tilde{k}$ is small (or both). The first condition holds if f_1 and f_2 are large; that is, if it is easy to accumulate knowledge. The second condition holds when c/π is small, that is, when the costs of undertaking a project are small relative to the potential (though not necessarily expected) profits.[13]

2.2 Forward-Looking Firms

Using the same model, we can analyze the relationship between number of firms and level of R&D when firms are not myopic, but make optimal decisions over long horizons. The model is otherwise unchanged.

Firm j does m_j projects. Each project is successful with probability $p(k_j)$. Firms

[13] Notice that the conditions under which dm/dn is positive are conditions under which entry into the industry would be easy. One would expect that entry would raise the value of c/π, and thereby eliminate the possibility of a positive relationship between M and n.

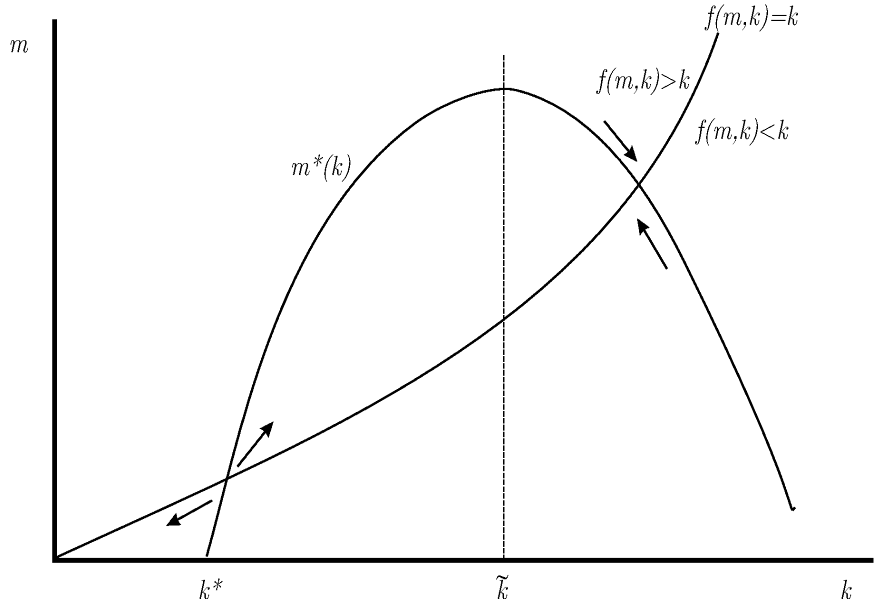

Figure 1: Diagram of Motion for Myopic Firms

For a fixed n, k^* is such that $p(k^*) = 1 - e^{-c/\pi}$ and $\tilde{k}$ is such that $p(\tilde{k}) = 1 - \exp\{-ce/\pi\}$. In general two steady states exist, with the larger value of k being stable. However, if the optimal m when $k = 0$, $m^*(0)$, is greater than 0, there is one intersection of optimal $m(k)$ and $k = f(k, m)$, and it is stable. If $m^*(0) < 0$ then there may not be any intersections, (if $m(k) < f(k, m)$ everywhere), in which case the steady state is at $k = 0,\ m = 0$.

are Bertrand competitors each period, that is, each period firms compete on price with any other firm that makes the same innovation. The probability of at least one of a firm's projects being successful is $q = 1 - (1 - p(k))^m$. In a symmetric equilibrium, where all firms have the same knowledge stock, the probability that no other firm has a successful project is $h = (1 - p(k))^{M-m}$ where M is the total number of projects undertaken by the industry. Thus the expected one-period profit of a firm is $EP = \pi hq - cm$ where c is the cost of undertaking a project. Again, for each firm, k evolves as $k_{t+1} = f(k_t, m_t)$, where $f(\cdot)$ is concave. Assume that knowledge depreciates quickly, so that $f_1 = 0$.

Firms maximize discounted expected profits, treating the sequence $\{h_t\}$ as fixed by the actions of the other firms. An equilibrium condition is that the prediction of the sequence $\{h_t\}$ (and equivalently the prediction of the sequence $\{M_t - m_t\}$) by any firm is realized. The firm's problem is written as

$$\max_{m_t} W(m_t, k_t) = \pi hq - cm_t + \gamma V(k_{t+1}) \tag{9}$$

where $V(\cdot)$ is the value function.

Differentiating, the first order condition is

$$\frac{\partial W}{\partial m_t} = \pi hq_m - c + \gamma \frac{\partial V(k_{t+1})}{\partial m_t} = 0. \tag{10}$$

By the envelope theorem:

$$\frac{\partial W}{\partial m_t} = [\pi hq_m - c]_t + \gamma\, [\pi hq_k dp/dk]_{t+1}\, f_2(k_t, m_t). \tag{11}$$

Symmetry implies that $M = mn$ and equilibrium implies that the predicted value of h is realized, so h becomes $h = (1 - p(k))^{M-m}$, and the first order condition can be written as

$$\frac{\partial W}{\partial m_t} = -\pi(1-p_t)^{m_t n}\ln(1-p_t) - c + \gamma\pi(1-p_{t+1})^{nm_{t+1}-1} m_{t+1} \frac{\partial p}{\partial k}(k_{t+1}) \frac{\partial k}{\partial m}(m_t) = 0, \tag{12}$$

or

$$c/\pi = -\left[(1-p)^{mn}\ln(1-p)\right]_t + \gamma\left[(1-p)^{nm-1} m \frac{\partial p}{\partial k}(k)\right]_{t+1} \frac{\partial k}{\partial m}(m_t). \tag{13}$$

which describes the relationship between m_t and m_{t+1}, determining the dynamics of this system. Again, in general, since $M = mn$, the relationship between M_t and M_{t+1} is not independent of n. Thus at finite times, the number of firms in the industry will have an effect on the amount of R&D performed. Without specifying functional forms, the direction of the effect is not determined. We move therefore to an examination of the steady state.

In the steady state $p_t = p_{t+1} = p = g(k)$; $k = f(k, m)$, and in equation 13, m_t $= m_{t+1}$.

Proposition 2: At least one steady state exists for sufficiently small c/π.

Proof: Treat the right hand side of the first order condition, equation (2), as a function of m, $RHS(m)$, noting that $m \in [0, \infty)$ and $p(m) \in [\overline{p}, 1)$ with $p(0) = \overline{p} \geq 0$.

$$\lim_{m \to \infty} RHS(m) = 0^+ \text{ since } \lim_{m \to \infty} \frac{\partial p}{\partial m} = 0. \tag{14}$$

If $\overline{p} > 0$ then $RHS(0) = -\ln(1 - \overline{p}) > 0$ and a steady state exists if $c/\pi < RHS(0)$.

Two steady states exist if $RHS(0) < c/\pi < \max RHS(m)$. If $\overline{p} = 0$ then $RHS(0) = 0$. But $\partial RHS/\partial m = p'(1 + \gamma)/n^2 > 0$. So for small enough c/π, two steady states will exist. †

Substituting the steady state values into the first order condition,

$$c/\pi = (1 - p)^M \left(-\ln(1 - p) + \gamma m p'/(1 - p)\right), \tag{15}$$

where again p' is dp/dm. Differentiating yields

$$\frac{\partial M}{\partial n} = \frac{m\left[Mp' \ln(1-p) + \gamma\left(\frac{-m(M-1)p'p'}{n(1-p)+mp''}\right)\right]}{\left[Mp' \ln(1-p) + \gamma\left(\frac{-m(M-1)p'p'}{n(1-p)+mp''}\right)\right] - (\ln(1-p))^2(1-p) + \gamma m p' \ln(1-p)}. \tag{16}$$

In general this is ambiguous in sign. If, however, both $M > 1$ and p'' is small, $\partial M/\partial n$ will be positive. A large value for M implies a large m and, since m and p are positively related, a large p. But because p is bounded above by 1, a large p implies a small p''. So if the steady state success probability is close to one, there is a positive relationship between industry R&D levels and the number of firms. At the other extreme, if there is a steady state solution at the minimum value of p, $\partial M/\partial n$ will be zero (since if $p = p_{min}$, $m = 0$). Whether the sign near $p = p_{min}$ is positive or negative is determined by the relationship between m and p.

3 An Example

This section presents a numerical example of the model, using particular functional forms for the probability of success as a function of knowledge, and for the evolution of knowledge. The welfare properties of this model are interesting in that there is an externality to doing R&D. If firm i increases its level of R&D, it affects firm j by reducing the probability that j will be uniquely successful in innovating. A planner can internalize this externality. The social planner's problem is not well-specified, however, unless more detail is given about the demand side of the market. This section presents the steady state solutions to both the firm's problem and to the planner's problem for the example discussed. The demand side is treated in three different cases—social benefits to innovation are either less than, equal to, or greater than benefits to a uniquely innovating firm.

Suppose the functional forms for the model are $k_{t+1} = \delta \ln(m_t + 1)$; and $p_t = \alpha - \beta e^{-k_t}$. By substituting, we can eliminate k and write $p_{t+1} = \alpha - \beta/(m_t+1)^\delta$, which implies that while $m \in [0, \infty)$, $p \in [\alpha - \beta, \alpha)$. As in the general case, the first order condition for the problem is

$$0 = [\pi h q_m - c]_t + \gamma \left[\pi h q_k\right]_{t+1} \frac{\partial p_{t+1}}{\partial m_t}. \tag{17}$$

In the equilibrium steady state, $c/\pi = (1-p)^M[-\ln(1-p) + \gamma m p'/(1-p)]$, so in this example

$$\begin{aligned} c/\pi = \quad & \left(1 - \alpha + \frac{\beta}{(m+1)^\delta}\right)^M \times \\ & \left[-\ln\left((1 - \alpha + \frac{\beta}{(m+1)^\delta}\right) + \frac{\gamma m \beta}{(m+1)^{\delta-1}(1-\alpha) + \beta(m+1)^{-1}}\right] \end{aligned} \tag{18}$$

Taking logarithms, n can be isolated to get

$$n = \frac{\ln(c/\pi) - \ln\left[-\ln\left(1 - \alpha + \frac{\beta}{(m+1)^\delta}\right) + \frac{\gamma m \beta}{(m+1)^{\delta-1}(1-\alpha)+\beta(m+1)^{-1}}\right]}{m \ln\left(1 - \alpha + \frac{\beta}{(m+1)^\delta}\right)}. \tag{19}$$

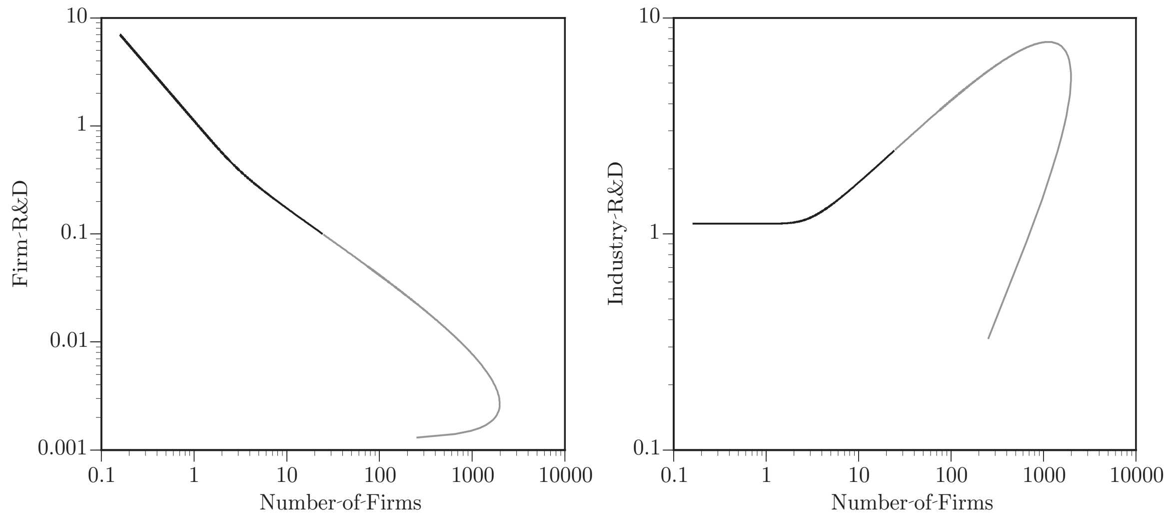

Figure 2: Steady state levels of Firm and Industry R&D.

Positive profits in black; negative profits in grey.

Using equation 19, the relationship between firm R&D levels and the number of firms in the industry can be described. The general form of this relationship is presented in Figure 2. In general there are two optimal values for m for every level of n, but at most one of these values corresponds to positive profits. Under the restriction that expected steady state profits are positive, each firm does less R&D as the number of firms increases. The general form of the relationship between steady state quantities of total industry R&D and the number of firms is presented in Figure 2. Here the multiple optima in m are translated into M, but again only some of the values, shown in black in the figure, represent positive profits for the firm. For all parameter values used, when expected steady state profits are restricted to be positive, the relationship between industry R&D and number of firms is positive; as the number of firms increases the total industry R&D also increases.[14]

In both figures, the curves represent points of both positive and negative profits.

[14] These results are in keeping with Li's (1999) summary of the earlier literature, a main finding of which "is that an increase in the number of firms increases *total* R&D spending but may decrease *per-firm* R&D spending." (Li, 1999, p. 385)

Points corresponding to positive profits are shown in black, negative profits in grey. Figure 2a shows the first order condition, equation 19 solved for firm R&D, m, as a correspondence with the number of firms, n. Figure 2b multiplies the firm level R&D, m, as generated from the first order condition by the number of firms to generate industry level R&D as a correspondence with the number of firms.

3.1 Comparative Statics

Several comparative statics experiments can be run using this example in order to judge the effect on R&D levels of the nature of the R&D process itself. The example has four parameters—α, β, δ, and π. Changing these parameters affects the level of R&D as a function of the number of firms, and the number of firms n^* such that expected profits in the steady state are zero.[15] Both of these are of interest in the comparative static experiments. The next paragraphs describe the effects of changing each of the four parameters.

The rents from innovating are equal to π. As the rents are increased, the marginal benefit to R&D increases for any level of R&D. This will evoke an increase in R&D by every firm. This reduces the probability of being a unique innovator, and thus the marginal benefit to R&D, due to the externality of R&D. For all levels of n, then, the relationship between π and M is positive, as expected. Further, when rents are higher, the industry can support more firms at a positive level of R&D; n^* increases.

The parameter δ measures the responsiveness of future knowledge to current R&D. The larger is δ, the bigger the increment to future knowledge from increasing R&D levels in the present. In the steady state, the *smaller* is δ, the smaller is the probability of success. This depresses expected profits for an individual firm, but that effect is more than offset by an increase in the probability of being uniquely successful. In response, a firm will increase its R&D, and so the relationship between

[15] This condition can be seen as the condition for endogenizing the number of firms.

industry R&D levels and the ease of knowledge accumulation, δ, is negative. On the other hand though, as it is easier to accumulate knowledge, (larger δ), more firms can be supported in the industry, and the level of R&D rises—the relation between δ and the supportable number of firms is positive.

The responsiveness of success probability to knowledge is determined by β. It has a second function here, though; it is the lower bound of the success probability. Reducing β reduces the scope for changing the probabilities by changing knowledge levels, ($p \in [\alpha - \beta, \alpha)$), and reducing β increases the probability of success at any knowledge level. Thus in general β and δ work in opposite directions. (The mathematical intuition for this is easy to see from equation 19: with the exception of the final term in the numerator, both β and δ only enter as $\beta(m+1)^{-\delta}$.) The relationship between β and M is positive. When β is close to α, that is, the minimum possible success probability is close to zero, the relationship between β and n^* is negative. When β is small, however, (approximately equal to $\alpha/4$), the relationship turns positive.

The maximum possible success probability is given by α, and, all else equal, the higher is α the higher is the probability of success for an individual firm. Again, there is the externality effect which causes the inverse relation between success probability and expected profits, so as α falls from one, the success probability falls, raising expected profits, to which firms respond by raising their levels of R&D. (Changing α places restrictions on β, since it must be that $\beta < \alpha$. This discussion assumes a constant α/β.) The relation between α and M is therefore negative. The relation between α and n^* is positive when α is near 1, but turns negative when α becomes sufficiently small. The value of α at which the relationship becomes negative decreases as π increases.

In general then, we should expect to see a high level of total R&D in industries in which rents from R&D are high; in which the probability of success is low; in which it is difficult to accumulate knowledge through the R&D process; but in which accumulating knowledge has a large impact on the absolute probability of success. The conditions describing industries in which it is possible for there to be large numbers of firms pursuing R&D are, with some qualification, the reverse—high rents, easy knowledge accumulation, low responsiveness of success probabilities to knowledge stocks, and high success probabilities.[16] If the zero-expected-profit condition determines the number of firms in an industry, then these conditions describe industries that will have many innovating firms. Thus empirically we would expect high levels of R&D at the industry level when many firms perform R&D. But this relationship is not driven directly, rather the causation is indirect, and the connection between the two variables, industry structure and R&D performance will be mediated, and controlled by several other industry-specific characteristics.[17] The conditions are complex, though, involving the conjunction of four factors, so empirical analysis would not be straight-forward.

3.2 Welfare Issues

The welfare properties of this model are interesting in that there is an externality to doing R&D. If firm i increases its level of R&D, it affects firm j by reducing the probability that j will be uniquely successful in innovating. A planner can internalize this externality. The social planner's problem is not well-specified, however, unless more detail is given about the demand side of the market. This section presents the steady state solutions to both the firm's problem and to the planner's problem for the example discussed. The demand side is treated in three different cases—social benefits to innovation are either less than, equal to or greater than benefits to a uniquely innovating firm.

Figure 3 presents a representative picture of the relationship between number of firms and industry R&D. In this figure it is assumed that the social value of the innovation is equal to π, the profits gained by a uniquely innovating firm. Clearly,

[16] This characterization assumes that the success probability when a firm has zero knowledge is small (β near α), and that the success probability when a firm has much knowledge is large (α near 1). When either of these conditions is violated, the characterization must be changed in line with the paragraphs above.

[17] This is consistent with the conclusion of Acs and Audretsch (1987).

if this is not the case, if the social value is greater than (less than) the private value, the social value curve will shift up (down) relative to the private value curve. Corner solutions are important since as the number of firms in an industry increases, the present value of the steady state to the representative firm decreases, becoming negative at a finite number of firms, n^*, in Figure 3. At this point, the optimal decision for a firm is to conduct no R&D.

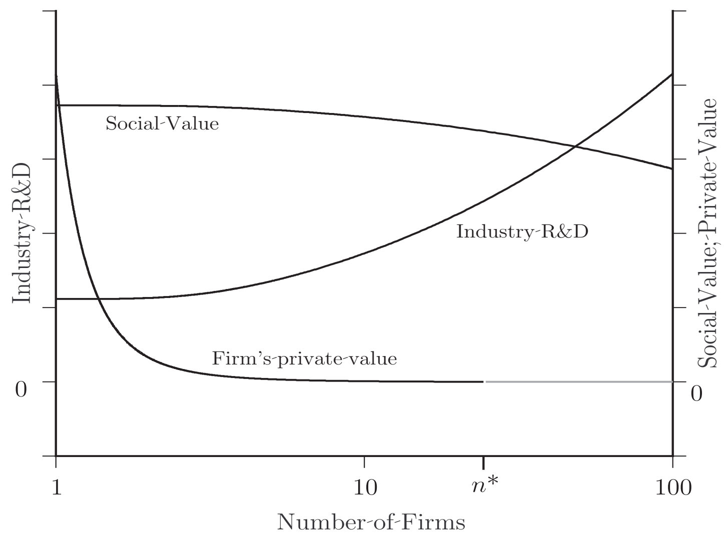

Figure 3: Steady state quantities as functions of the number of firms

The data represented here are from the example presented in the previous section. The number of firms can be endogenized by the zero profit condition: $n^* \approx 25$.

The R&D of a single firm creates an externality in that if one firm increases its level of R&D, it lowers expected profits for every other firm by reducing the probability that the other firm will be uniquely successful. When a planner internalizes this externality, however, the first order conditions for the firm and the planner are identical, with the exception that π, the profits to a uniquely innovating firm, are replaced by Z, the social benefits from having the innovation.[18] Thus if $Z = \pi$, the socially optimal number of projects is equal to the number that the market would

[18] Sah and Stiglitz find this result in their static model. In the current model the first order conditions for the planner are $c/Z = (1-p)^M \ln(1-p)$ if the planner is myopic (see equation 6,

pursue.[19] In general, if $Z > \pi$, the market under-supplies R&D, if $Z < \pi$, the market over-supplies.

There is one qualification to this general conclusion. Since the firm only receives profits if it is the only innovator, as the number of firms increases the expected profits of each firm declines. This consideration does not apply in the planner's problem, since no matter which firm (or even how many firms) innovates, the same social benefits accrue. This implies that the market can be driven to a corner solution with no firm doing R&D, when the social optimum is for there to be positive R&D. When internalizing the externality, the discounted value of the steady state goes to zero as the number of firms increases, but much more slowly, which implies that the socially optimal level of R&D can be positive even when there are large numbers of firms in the industry. There is an upper bound to the feasible number of firms in the industry, even under the planner's solution, given the assumption that firm R&D is bounded below: $m \geq \epsilon > 0$, due to the fixed cost of R&D, c. Again not shown in Figure 3 is the negative relation between individual firm R&D levels and the number of firms—as the number of firms grows indefinitely, the amount of R&D per firm becomes very small.

As mentioned above, the exact location of the curves in Figure 3 will depend in part on how the social gain to the innovation compares with the gain to the uniquely innovating firm. Using the case in which the two quantities are the same as a benchmark, the social value curve shifts up as the social gain increases and the socially optimal quantity of R&D would lie everywhere above the current industry R&D curve. As expected, if the market under-values innovation, it will also under-supply it.

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and $c/Z = -\left[(1-p)^{mn}\ln(1-p)\right]_t + \gamma\left[(1-p)^{nm-1}m\frac{\partial p}{\partial k}(k)\right]_{t+1}\frac{\partial k}{\partial m}(m_t)$ if he is forward-looking (see equation 13.

[19] It would seem that the planner should want the same amount of R&D no matter how many firms there are. That this is not so stems from the production of knowledge. The same amount of R&D with more firms implies that each firm has less knowledge next period thereby reducing the probability of innovation. If knowledge accumulation were a function of total R&D, M, rather than of firm R&D, M/n, this argument would not go through, and the number of firms would be irrelevant.

4 Discussion

When R&D is speculative and the success of projects is not certain, but where the probability of success is affected by past R&D, there is a relation between number of firms and R&D. As argued above, and as suggested by the example, in general one should expect that the relationship between number of firms and total R&D is positive. The dynamics are such that more R&D this period implies more knowledge next period. This has two effects. First, it raises the probability that a firm will succeed in its R&D; but second, it implies that rival firms are also more likely to succeed. The assumption of Bertrand competition is crucial here. The success of a rival has a devastating impact on profits, and this possibility will cause reductions in firm level R&D. In principle these reductions can be sufficient to lower industry level R&D as the number of firms increases. Numerical calculations show, however, that this outcome is not typical. While increasing the number of firms does reduce the amount of R&D done by each firm, it increases total industry R&D, up to a point. As the number of firms increases, the present value of the steady state to the representative firm declines, and eventually turns negative, at which point, clearly, the solution moves to the corner, with no firms doing any R&D.

The planner's problem generates very similar results. Because the planner internalizes the external effects caused when a single firm increases its R&D, thereby lowering the expected profits of other firms, the plan runs into corners at a much larger number of firms. The social value of the steady state stays positive longer, and falls with the number of firms. As in the market solution, industry R&D increases with the number of firms.

It is worth pointing out that the direction of the relationship between industry size and industry R&D depends crucially on the mechanism that distributes rents. In this model, rents are distributed *ex post* to the winner of the patent race. It is possible, though, that rents from successful R&D could be distributed differently in a different situation. Consider a case in which R&D is more like product development and less like research. Here, success of an R&D project is guaranteed if it receives the required resources. If one of those resources is knowledge, and if knowledge is a public good within a firm, then the results described above are reversed. If firms are Bertrand competitors, they will not duplicate each other's R&D, as that would lead to losses for all firms. Rather, in equilibrium, the potential R&D projects will be "distributed" *ex ante* with each project being pursued by one firm (though a single firm may do more than one project). Taking the supply of potential projects as exogenous, more firms means fewer projects per firm, and less knowledge next period. Less knowledge implies that the "difficult" projects are now impossible, and so fewer projects are feasible next time. The relation between number of firms and total R&D is negative.[20]

The opposition of the results of the formal model with those just described stems from the competitive structure of the industry. In the situation just sketched, all R&D, if undertaken with the required resources, yields a positive return (think of it as creating a differentiated product). Knowledge is a "good thing", since more knowledge implies that it will be possible to successfully complete more "difficult" projects. The distribution of rents from R&D would be determined by the *ex ante* distribution of R&D projects among firms in equilibrium. This is not the case in the formal model. Firms directly compete with their R&D projects, and the

[20]It is possible that the relationship is positive if returns are much greater than costs (c/π is small) and when knowledge accumulates quickly, and therefore losses now, from unprofitable R&D projects, translate into large future gains through making more difficult projects possible. Adding a firm to an industry always reduces the number of projects undertaken by each firm, which has a knock-on effect, through decreasing the number of feasible projects in the future. (It has a second effect, of course, namely that the profits from the profitable projects will be divided among more firms.) If a small increase in R&D today generates a large increase in tomorrow's knowledge though, firms reduce their R&D only slightly in response to an additional firm, and so the net effect of increasing the number of firms is to increase total R&D. Within the confines of the model this is a possible outcome, but looking outside the model permits an argument that this is unlikely. Three factors contribute in creating such a situation in which more firms implies more R&D: costs of R&D relative to returns are small; knowledge accumulates quickly as R&D is performed; and as the knowledge base of a firm increases, the number of profitable projects increases rapidly. But these three factors also create a situation in which entry into the industry is likely. The costs of acquiring a competitive knowledge base are low (costs of performing R&D are low and knowledge accumulates quickly as one does so), and, once that base is acquired, profits from participating in the industry are high. A significant amount of entry, though, would increase the costs of R&D relative to the benefits, and so destroy the conditions generating the positive relationship between number of firms and industry R&D. This argument suggests that in general we would expect that more firms implies less R&D.

success of one project implies zero or negative profits for all the other projects in the industry. In this repeated patent race setting, distribution of rents is determined by the outcome of the race. This generates an externality to R&D, which complicates the relationship between current R&D, future knowledge, future expected profits and number of projects. Knowledge here is not necessarily a good thing, since by increasing the probability that rivals will succeed, more knowledge can lower the expected profits of a firm.

The R&D process is driven to a great extent by the accumulation and use of knowledge. Recognizing the importance of knowledge and the dynamic nature of its accumulation generates rich results in the study of R&D. Whether more knowledge is a good thing, however, depends crucially on the nature of the research, and on the form of competition in the industry in which the R&D is taking place.

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