THE INFLUENCE OF SPILLOVERS, PRODUCT DIFFERENTIATION AND ENTRY ON TECHNOLOGICAL CHANGE

Hans van Meijl

(Maastricht, August 1994)

MERIT
Maastricht Economic Research Institute on Innovation and Technology
University of Limburg P.O. Box 616 6200 MD Maastricht
Netherlands tel: 31 43 888741 fax: 31 43 216518

TABLE OF CONTENTS

1. Introduction 1

2. Derivation of Demand Functions with Product Differentiation 2

3. Cost Reducing and Demand Creating R&D with Inter and Intra-industry R&D Spillovers 4

   3.1 Efficiency Improvements and their influence 4
   3.2 Quality Improvements and their influence 6

4. An Endogenous Technological Change Model 8

5. Steady State Growth Rates 12

6. Dynamics 13

7. Steady State 21

   7.1 R&D Intensities 21
   7.2 R&D Levels and Spillover Effects 24
   7.3 Cost Reducing and Demand Creating Innovations and Spillovers 27
   7.4 Cost Reducing and Demand Creating Innovations and Entry 27
   7.5 Welfare, Entry and Spillovers 30

8. Conclusion 34

Appendix A: "Derivation of Demand Functions" 36

Appendix B: "Calculation of Steady State Growth Rates" 38

Appendix C: "Time Elimination" 42

Appendix D: "Checking Saddle Point Stability" 47

Appendix E: "Values of Parameters and Exogenous Variables used in Base Run Scenario" 49

References 50

1 Introduction

Firms perform R&D to improve their competitiveness. Benefits occur in the shape of cost-reducing and demand-creating innovations. But these benefits are imperfectly appropriable. A part of the knowledge generated by one firm can be used also by other firms. The R&D of a firm improves therefore not only its own technological level but also the technological level of its competitors. The degree of appropriability therefore influences a firm's incentives to R&D because it knows that a part of its own efforts will be used by other firms and it can use results of other firms R&D. Spence (1984), in particular, finds that a decrease in appropriability, creating larger spillovers, reduces the incentives of individual firms to invest in R&D. Many other authors have examined the relation between incentives for R&D investment and the degree of appropriability in a static context[1]

As Levin and Reiss (1988) we assume that firms can perform cost-reducing process R&D to increase the efficiency level of the production process and perform demand creating product R&D to increase the perceived quality of a product to increase expected demand[2]. Just as in Van Meijl and Van Zon (1993) we will develop a dynamic model to take account of the intertemporal benefits of R&D. The Van Meijl and Van Zon model will be extended with inter- and intraindustry spillover effects, product differentiation and entry. Another difference with the mentioned studies is that in this model quality improvements may also increase unit production costs.

With regard to expected demand we assume that people are not only interested in the quantity of a good but also in the quality of a good. We also distinguish between "Love of Variety" and "Good Characteristics" preferences. The difference between these preferences is that love-of-variety-people value variety in its own right (Spence (1976), Dixit and Stiglitz (1977)). These two kinds of preferences imply different demand characteristics and influence the decision process of a firm with regard to technological investments.

Another important determinant of the technological and economic behaviour of a firm is entry. The number of firms limits the appropriability of R&D but it also increases the industry knowledge stock. Further entry has an important influence on the perceived price and quality demand elasticity. The influence of entry on welfare is of course very dependent on the kind of consumer preferences.

This paper investigates the influence of spillover effects, consumer preferences, and entry on the technological performance and profit level of individual firms and the general welfare level. Another important aim is to

[1]For example, Loury (1979), Lee and Wilde (1980), Levin and Reiss (1988), Cohen and Levinthal (1989), De Bondt et al (1992).

[2] The perceived quality level is the quality level of a product relative to the quality level of similar/competing products.

explore the relation between process and product R&D. Are these two kinds of R&D mainly substitutes or do they reinforce each other?

In section two we derive the demand functions which display "Love of Variety" or "Good Characteristics" preferences. Section three describes the technology generation functions and the modelling of spillovers. Section four describes the total endogenous technological change model and the characteristics for any potential equilibrium. The fifth part discusses the steady state growth rates. Part six shows the dynamic behaviour of the model and part seven discusses the influence of spillovers, entry and product differentiation on the steady state R&D intensities, technological performance and welfare.

2 Derivation of Demand Functions with Product Differentiation

What is important in our model is that consumers value not only the quantity but also the quality of a good. They enjoy utility of the amount of characteristics which are present in a good. We consider the situation in which there are two kinds of goods: differentiated products and homogenous products. Consumer preferences are represented by a two level utility function:

$$U=[u_1(D),u_2(H)] = \frac{X_0}{\zeta}.D^{\zeta} + H \ , \qquad \zeta < 1 \tag{2.1}$$

where u_2(H) is the subutility function of the homogenous good (H) and u_1(D) the subutility function of the differentiated good (D). We assume that the utility of the homogenous good depends only on the quantity of the good consumed (u_2(H)=H) and the utility of the differentiated good depends on the quantity and quality of each variety consumed. The overall utility function (U[.]) adds the two sectoral utility levels.

The utility function of the differentiated good is an elaborated Spence-Dixit-Stiglitz utility function (Spence (1976), Dixit and Stiglitz (1977)). A quality index for each good is included.

$$D = \left[\sum_{i=1}^{n} (y_i.Q_{p,i}^{b})^{\rho}\right]^{\frac{1}{\rho}} \qquad , \qquad a = \frac{1}{1-\rho} > 1 \tag{2.2}$$

where D is the quality characteristic index, y_i is the consumption of good i, $Q_{p,i}$ is the perceived quality level of good i, n is the number of varieties of a certain good and a is the elasticity of substitution between differentiated products[3]. The perceived quality level of a product is the result of a firms decision process which will be described in sections three and four. The equilibrium quality

[3] The substitution elasticity (a) has to be larger than one so that the price elasticity of demand, perceived by a firm, will be larger than one (see equation 4.2). This is required to avoid negative marginal revenue in a monopolistic situation.

levels will be equal across sectors, $Q_{p,i}=Q_{p,j}$, because we assume that each firm confronts the same decision problem and that we study only symmetric equilibria.

It is easy to show that this elaborated Spence-Dixit-Stiglitz subutility function contains love of variety. Assume that the expenditures for the differentiated good are I_D. In the symmetric case are quantities and prices of all varieties equal. The demand for one variety is therefore $y_i=I_D/(n.p_i)$. The level of the quality characteristic index (D) obtained from I_D in the symmetric case is

$$D = \left[n.Q_{p,i}^{b.\rho}.\left(\frac{I_D}{n.p_i}\right)^{\rho}\right]^{\frac{1}{\rho}} = n^{\frac{1-\rho}{\rho}}.Q_{p,i}^{b}.\frac{I_D}{p_i} = n^{\frac{1}{a-1}}.Q_{p,i}^{b}.\frac{I_D}{p_i} \qquad a>1 \tag{2.3}$$

The index D depends positively on the number of varieties (n). Equation (2.1) shows that a higher index D provides a higher utility level. The utility level increases therefore as the number of varieties increases. This confirms that the demand specification allows for *"Love of Variety"*. The love of variety effect declines when the elasticity of substitution between pairs of varieties (a) increases. The influence of the number of different varieties (n) on D and therefore the utility level becomes neglible when this elasticity (a) becomes large. People derive then only utility from the quality level ($Q_{p,i}$) times the total number of goods (I_D/p_i) bought (the number of product varieties plays no role): the total amount of *"Good Characteristics"*. Variety is not valued per se anymore but people are only interested in the good characteristics.

The utility maximization problem of the consumer can be solved in a two-stage budgeting procedure (Dixit and Stiglitz, 1977)[4]. This procedure is described in appendix A. The demand function for each differentiated good implied by the utility function, equation (2.1), and the quality characteristic index, equation (2.2), is given by equation (A.15).

$$y_i = X_0^{\varepsilon}.Q_{p,i}^{b.(a-1)}.p_i^{-a}.P_D^{a-\varepsilon} \tag{2.4}$$

where $a=1/(1-\rho)>1$ and $\varepsilon=1/(1-\zeta)>1$.
The definition of P_D is stated in equation (A.16)

$$P_D = \left[\sum_{i=1}^{n} Q_{p,i}^{b.(a-1)}.p_i^{1-a}\right]^{\frac{1}{1-a}} \tag{2.5}$$

The elasticity of substitution is given by parameter a and the overall price elasticity of demand is given by ε. It is logical to assume that the elasticity of substitution within a sector is larger than the overall price

[4]The two stage maximisation process is allowed because the separability of the total utility function (equation (2.1)) and the linear homogeneity imposed by equation (2.2).

elasticity of demand (a>ε). The demand of a differentiated product will depend positively on the quality level and the overall price index and negatively on its own price level.

In the symmetric equilibrium the demand function for each variety is

$$y = X_0^{\varepsilon}.(Q_p)^{b.(\varepsilon-1)}.(p)^{-\varepsilon}.n^{\frac{a-\varepsilon}{1-a}} \tag{2.6}$$

3 Cost Reducing and Demand Creating R&D with Inter and Intra-industry R&D Spillovers

3.1 Efficiency Improvements and their Influence

The influence of efficiency improvements is straight forward in the model: it increases the productivity of the production process. We assume a linear homogenous production function

$$y_i = A_i.F(K_i,L_i) = A_i.K_i^{\alpha}.L_i^{1-\alpha} \tag{3.1}$$

where y_i is output of firm i, K_i is capital input of firm i, L_i is labour input of firm i and A_i is the total factor productivity or efficiency of production process of firm i.

In the first place, firms engage in process R&D to improve the efficiency of their production process (Dasgupta and Stiglitz 1980, Sato and Suzawa 1982). One important characteristic of performing R&D is that the benefits are not perfectly appropriable by the firm. A part of the knowledge generated by own R&D spills over to other firms and industries. Just like Levin and Reiss (1988) we assume therefore that the efficiency of the production process is not only influenced by own process R&D but also by process R&D of other firms in the same industry and process R&D of other industries[5]. The "use" of R&D from firms operating in the same industry is called an intra-industry R&D spill-over effect and the "use" of R&D from firms operating in different industries is called an inter-industry spill-over effect.

We elaborate the studies mentioned before by putting this process in a dynamic context and assuming that the efficiency level itself has a positive influence on the productivity of R&D. The current efficiency level can be seen

[5]R&D from other industries can only be obtained from industries which also produce differentiated products. In the simple utility function of section two (equation (2.1)) we assumed only one differentiated good which make inter-industry spill-overs impossible. But the separability condition of this utility function, our specifications of the product differentiation process (equations (2.2)) and the presence of the homogenous good make it possible to introduce more differentiated goods without affecting the demand functions of the differentiated good already present.

as the result of process R&D done in the past and therefore as some knowledge stock. A higher knowledge stock implies a higher productivity of the R&D process[6]. Our specification of the productivity generation process is chosen in such a way that it contains the specifications of earlier contributions to this field as special cases

$$\frac{dA_i}{dt} = (\eta . A_i^{m_a} . T_{c,i}^{\theta_1} . S_c^{\theta_2}) . R_{c,i}^{\theta} \qquad 0 \leq \theta, \theta_1, \theta_2 \leq 1 \ , \quad m_a < 1 \tag{3.2}$$

The own level of process R&D ($R_{c,i}$) is the principal determinant of the change in the productivity level (dA_i/dt) of firm i. The other three variables, i.e. the intra-industry pool of knowledge ($T_{c,i}$), the inter-industry pool of knowledge (S_c) and the productivity level itself (A_i), influence the productivity of the own process R&D. We assume diminishing returns to own R&D and the two pools of R&D. The inter-industry pool of knowledge is assumed to be exogenous and the intra-industry pool of knowledge can be modelled as[7]

$$T_{c,i} = R_{c,i} + \omega_a . \sum_{j \neq i}^{n} R_{c,j} \tag{3.3}$$

ω_a being the extent of process R&D spillovers: i.e. the part of rival R&D you can use. It is important to distinguish this effect with the productivity of process R&D spillovers, represented by θ_1 in equation (3.2), the effectiveness of rival R&D (see Levin and Reiss 1988).

Several special cases of this productivity function are used in previous static work. By considering only the influence of own process R&D, $\theta_1 = 0$, $m_a = 0$ and $\theta_2 = 0$ one obtains the Dasgupta and Stiglitz (1980) specification. When $\theta = 0$, $m_a = 0$, $\theta_2 = 0$ and $\theta_1 = 1$ we get the Spence (1984) specification which is also used by the Bondt et al.(1992). The Levin and Reiss specification which is most similar to the model specification can be obtained when $m_a = 0$ and $\theta_2 = 0$. The specification in the dynamic Sato and Suzawa (1983) analysis will be obtained when $\theta_1 = 0$, $\theta_2 = 0$ and $m_a = 1$.

We have two critical remarks on earlier specifications of the productivity generation process. First, Levin and Reiss (1988) include separately own R&D ($R_{c,i}$) and a pool consisting of own and rival R&D ($T_{c,i}$) which causes double counting of own R&D. Their argument to justify this specification is that in this manner they "emphasize that own R&D contributes to a firm's idiosyncratic capabilities as well as to an industry pool of knowledge (Levin and Reiss, p. 540). This implies that when a firm engages in own R&D it creates new knowledge which increases its productivity level which is logical. But the double counting method implies that when the same new knowledge is added to the general existing knowledge pool it increases the productivity level of the

[6]In the endogenous growth theories is also assumed that the knowledge stock adds to the productivity of the research sector (see, e.g. Romer (1990), Grossman and Helpman (1991a, 1991b) and Aghion and Howitt (1990). Similar reasons in the context of human capital accumulation can be found in Lucas (1988).

[7]This way of modelling the knowledge stock is also used in Spence (1984) and Levin and Reiss (1988).

firm once again although there is no new knowledge available to the firm. We will include this "double counting" method in our specification of the productivity generation process to study its influence on the model results. We can also avoid this double counting by putting the parameter θ equal to zero.

Second, in other dynamic models such as Sato and Suzawa (1983) it is assumed that m_a is equal to 1[8]. This implies that the same level of R&D expenditures is required to achieve a certain growth rate of the productivity level no matter if one possesses a low or a high productivity level. But is it not more intuitive to assume that more resources are needed if one possesses a higher productivity level.

$$\frac{\frac{dA_i}{dt}}{A_i} = \eta . f(R_{c,i}, T_{c,i}, S_{c,i}).A^{-\beta_a} \qquad \beta_a > 0 \qquad \textbf{(3.4)}$$

$$\frac{dA_i}{dt} = \eta . f(R_{c,i}, T_{c,i}, S_{c,i}).A^{1-\beta_a} \qquad m_a = 1 - \beta_a < 1$$

3.2 Quality Improvements and their Influence

Perceived quality improvements are mainly done to increase the expected demand. Section 2 showed us that when quality is positively valued by consumers it has a positive influence on the demand functions. Quality improvements are changes in the real characteristics of a good it can therefore be expected that a higher quality level increases unit production costs. Dorfman and Steiner (1954) state that

"By quality we mean any aspect of a product, including the services included in the contract of sales, which influences the demand curves. The essential difference from advertising is that changes in quality enter into variable costs."

When we take into account the variable cost increasing effect of quality

[8] This is also an assumption which is characteristic of the "new" growth theories.

we have to adjust our production function described in section 3.1[9].

$$y_i = A_i . L_i^{1-\alpha} . K_i^{\alpha} . Q_{p,i}^{-\xi} \qquad \textbf{(3.5)}$$

This cost increasing effect of quality is neglected in Levin and Reiss (1988) and Van Meijl and Van Zon (1993).

We treat the perceived quality generation process in much the same way as we treated the productivity generation process. Perceived quality improvements can be created by engaging in own product R&D ($R_{d,i}$). The productivity of own product R&D is dependent on the intra-industry pool of product knowledge ($T_{d,i}$) and the inter-industry pool of product knowledge ($S_{d,i}$) and dependent on the quality level itself ($Q_{p,i}$), which can be seen as the knowledge stock of product R&D. The quality generating process is[10]

$$\frac{dQ_{p,i}}{dt} = \frac{\gamma . Q_{p,i}^{m_q} . S_d^{\delta_2} . T_{d,i}^{\delta_1} . R_{d,i}^{\delta}}{W_{d,i}^{\delta_3}} \qquad \textbf{(3.6)}$$

The inter-industry knowledge stock is again exogenous to the firm and the intra-industry knowledge stock will be constructed in the same manner as in the case with process R&D. It contains own product R&D and a fraction of the product R&D of all the other firms. This fraction, ω_q, symbolizes the "extent" of the quality spillover effect.

---

[9] The general idea here is that with a certain amount of inputs (labour and capital) you can produce a certain amount of quality characteristics: the number of goods produced (y_i) times the unit quality level ($Q_{p,i}=Q''_{p,i}/y_i$, where $Q''_{p,i}$ is the total number of quality units). There are constant returns to scale with respect to the number of products produced (y_i) and decreasing returns with respect to the unit quality level. To achieve a two times as high unit quality level you have to increase your total amount of production factors per unit output with more than two times. Assuming a linear homogenous Cobb Douglass function

$$Q_{p,i} = \frac{Q_{p,i}^{//}}{y_i} = \left[F\left(\frac{L_i}{y_i}, \frac{K_i}{y_i}\right) \right]^{\frac{1}{\xi}} = \left[A_i . \left(\frac{L_i}{y_i}\right)^{1-\alpha} . \left(\frac{K_i}{y_i}\right)^{\alpha} \right]^{\frac{1}{\xi}} \quad , \quad 0 < \frac{1}{\xi} \leq 1$$

The production function based on this condition is

$$y_i = A_i . K_i^{\alpha} . L_i^{1-\alpha} . Q_{p,i}^{-\xi}$$

[10] When product R&D is interpreted as advertising and the quality level as goodwill this function contains several specifications which have been used in previous studies. The Nerlove and Arrow (1962) specification can be obtained when $m_q=\delta_1=\delta_2=\delta_3=0$ and $\delta=\gamma=1$. The Gould (1970) specification differs from this Nerlove and Arrow specifications in that $\delta \neq 1$.

$$T_{d,i} = R_{d,i} + \omega_q . \sum_{j \neq i}^{n} R_{d,j} \qquad \textbf{(3.7)}$$

This situation is again characterised by a double counting of own product R&D. Again we will study the influence of this double counting method with the situation which avoid double counting by setting δ equal to zero.

But at this point the similarity with the productivity generation process ends. As Levin and Reiss (1988), we use perceived quality, which is the quality level of a product relative to the quality level of similar / competing products. An increase in the product R&D level of competing products may therefore reduce the perceived attractiveness of a firm's product. Levin and Reiss catch this effect by assuming that the intra industry spillover pool, own R&D and a part of other firms R&D ($T_{d,i}$), is also the relevant pool of available knowledge which has a negative influence on the perceived-quality generation process. The parameter δ_1 can therefore be positive or negative in their model. But the relevant knowledge pool which threatens (reduces) your perceived quality level is not your own R&D and a part of other firms R&D but other firms R&D and a part of your own R&D. The threatening intra industry knowledge pool, $W_{d,i}$, can therefore be defined as

$$W_{d,i} = \sum_{j \neq i}^{n} \left[R_{d,j} + \omega_q . \sum_{i \neq j}^{n} R_{d,i} \right] \qquad \textbf{(3.8)}$$

4 An Endogenous Technological Change Model

Consider a firm in a differentiated product industry that chooses its price level, productivity level and quality level so as to maximize the present value of its profits. It can influence its productivity level by engaging in process R&D and its quality level by engaging in product R&D. The intertemporal profit maximisation for a firm is[11]

[11] In stead of the Cobb Douglas production function itself we use the related total variable cost function. The advantage of this total variable cost function approach in comparison with a direct production function approach is that the dual cost function approach automatically implies the optimum allocation of labour and capital. In this way we can save two control variables, capital and labour, see Sato and Suzawa (1983).

$$\underset{p_i,\ R_{c,i},\ R_{d,i}}{Max}\ \pi_i(0) = \int_0^{\infty} e^{-rt}\left[p_i.y_i(p_i,Q_{p,i})-TC_i(y_i,A_i,Q_{p,i})-R_{c,i}.q_c-R_{d,i}.q_d\right]dt$$

$$s.t.\quad y_i=X_0^{\varepsilon}.Q_{p,i}^{b.(a-1)}.p_i^{-a}.P_D^{a-\varepsilon} \qquad with \quad a>\varepsilon>1$$

$$P_D = \left[\sum_{i=1}^{n} Q_{p,i}^{b.(a-1)}.p_i^{1-a}\right]^{\frac{1}{1-a}}$$

$$TC_i = \frac{Q_{p,i}^{\xi}}{A_i}.y_i.w^{1-\alpha}.v^{\alpha}.(1-\alpha)^{\alpha-1}.\alpha^{-\alpha} \qquad with \quad 0\leq\alpha\leq1$$

$$dA_i/dt=\eta.A_i^{m_a}.R_{c,i}^{\theta}.T_{c,i}^{\theta_1}.S_{c,i}^{\theta_2} \qquad with \quad 0\leq\theta,\ \theta_1,\ \theta_2,\leq1\ ,\ m_a<1$$

$$dQ_{p,i}/dt=(\gamma.Q_{p,i}^{m_q}.R_{d,i}^{\delta}.T_{d,i}^{\delta_1}.S_{d,i}^{\delta_2})/W_{d,i}^{\delta_3} \qquad with \quad 0\leq\delta,\ \delta_1,\ \delta_2,\ \delta_3\leq1\ ,\ m_q<1$$

$$A_{t=0} = A_0 \quad , \quad Q_{p,t=0} = Q_{p,0} \qquad \mathbf{(4.1)}$$

where w is the wage level and v is the user cost of capital. The presence of subscripts c or d in a variable implies that this variable is related with respectively process R&D and product R&D. The subscript a in a parameter indicates a relation with productivity and the subscript q symbolises a relation with quality.

It will be assumed that the firm takes as given the price and R&D strategy of other firms. The analysis is limited to the symmetric case: costs, technological opportunities, initial quality and productivity level and demand conditions for each firm within an industry are equal. The number of firms (n) is exogenous, which enables us to investigate the effects of market concentration on technological progress and welfare. Further, we assume that wages (w), the user cost of capital (v), the process- and product R&D prices (resp. q_c and q_d), the inter-industry process- and product R&D stocks (resp. $S_{c,i}$ and $S_{d,i}$) and the autonomous scale of demand (X_0) grow with a constant exogenous growth rate.

Optimal control theory is used to solve this problem. The state variables are a firms' productivity level (A_i) and its quality level ($Q_{p,i}$) and the choice variables are its pricelevel (p_i), process R&D level ($R_{c,i}$) and product R&D level ($R_{d,i}$). The solution of the current value Hamiltonian associated with this problem is given in appendix (B) and appendix (C). In appendix (B) we calculate the steady state growth rates of the system.

Characteristics of any potential equilibrium

First, marginal costs have to be equal to marginal revenue for the three choice variables. With respect to the price level are marginal costs equal to the "perceived" marginal revenue (see equation (B.3) and equation (B.4) in appendix B)[12].

$$\left(1-\frac{1}{a_p}\right).p_i = \frac{TC(A_i,Q_{p,i})}{y_i} \quad ; \quad a_p = a-\left(\frac{a-\varepsilon}{n}\right)>1 \qquad \textbf{(4.2)}$$

where a_p is the perceived price elasticity by a firm. It is called the perceived price elasticity because it is calculated under the assumption that other firms keep their price and R&D levels fixed, which does not have to be the case. The perceived price elasticity increases with the number of firms/varieties, the elasticity of substitution between varieties within an industry and the general price elasticity of demand. It approaches the elasticity of substitution (a) as the number of firms becomes large and it is equal to the general price elasticity (ε) in a monopolistic situation.

The price will be determined as a markup over marginal costs. The specification of the production function implies that marginal costs are independent of the production level and equal to the variable production costs. The margin in excess over unit costs can therefore be used to cover fixed product and process R&D costs. The markup is only dependent on the perceived price elasticity.

The static marginal cost-is-equal-to-marginal revenue conditions for process and product R&D are calculated in equation (B.5) and equation (B.7), respectively.

$$q_c = \mu.\frac{\partial(dA_i/dt)}{\partial R_{c,i}} = \mu.\frac{dA_i/dt}{R_{c,i}}.\left[\theta+\frac{\theta_1}{1+\omega_a.(n-1)}\right] \qquad \textbf{(4.3)}$$

$$q_d = \lambda.\frac{\partial(dQ_{p,i}/dt)}{\partial R_{d,i}} = \lambda.\frac{dQ_{p,i}/dt}{R_{d,i}}.\left[\delta+\frac{\delta_1}{1+\omega_q.(n-1)}-\frac{\omega_q.\delta_3}{1+\omega_q.(n-1)}\right] \qquad \textbf{(4.4)}$$

where μ and λ are the co-state variables of the optimal control problem. They can be interpreted as the marginal value of respectively $Q_{p,i}$ and A_i or the increase in future profits resulting from an increase in respectively $Q_{p,i}$ and A_i at current time t. With this in mind it is easy to see that the right hand side of these two equations depicts the marginal profits of R&D in the future, while the left side represents current marginal R&D costs. The expressions between the brackets in respectively equation (4.3) and equation (4.4) represent the R&D elasticities of respectively process and product R&D in the symmetric equilibrium. It is important to note the different influence on these elasticities

[12] Helpman and Krugman (1989, p.90) describe the perceived marginal revenue as "*perceived marginal revenue*-the increase in revenue that a firm expects to receive by producing one more unit, which is always less than the price (because of the effect on intramarginal sales) but may exceed the true marginal revenue that would prevail if the industry acted in concert".

of the *extent* (ω_a, ω_q) and *productivity* (θ_1, δ_1, δ_3) of spillovers. The extent of spillovers and the productivity (δ_3) of the threatening knowledge pool ($W_{d,i}$) have a negative influence on these elasticities whereas the productivities (θ_1, δ_1) of the intra industry spillover pool ($T_{c,i}$, $T_{d,i}$) have a positive influence on these elasticities.

If we compare the elasticity of product R&D with that in the case of the Levin and Reiss specification, in which $W_{c,i}$ is equal to $T_{c,i}$, the elasticity of product R&D would be $[\delta+(\delta_1-\delta_3)/(1+\omega_q.(n-1)]$. With regard to this threatening intra-industry spillover pool our specification recognises that only a part of our own R&D will be used by other firms.

The dynamic conditions which describe the development of the marginal value of A_i and Q_{pi} are given by equation (B.8) and equation (B.11).

$$-\frac{d\mu}{dt} = \frac{TC_i}{A_i} - \mu.(r - m_a.\hat{A}_i) \tag{4.5}$$

$$-\frac{d\lambda}{dt} = \frac{TC_i}{Q_{p,i}}.(b-\xi) - \lambda.(r - m_q.\hat{Q}_{p,i}) \tag{4.6}$$

The marginal value of the productivity and quality level depreciates at the rate at which productivity and quality are contributing to the current profits (this is represented by the first term at the right hand side in both equations). The marginal value appreciates at the rate of the marginal opportunity costs of investing in productivity and quality (this is represented by the second term at the right hand side in both equations)[13].

The perceived quality elasticity of demand (b_p) is calculated in equation (B.10):

$$b_p = b.(a-1) - \frac{(a-\varepsilon).b}{n} \qquad b.(\varepsilon-1) \le b_p < b(a-1) \tag{4.7}$$

The first term on the right hand is the direct influence of quality on demand and the second term is the influence of quality on P_D; the indirect effect on demand. An interesting feature of this statement is that the perceived quality elasticity increases as the number of firms increases. A higher perceived quality elasticity means that the benefits of investing in quality improvements are perceived to have a larger influence on expected demand. This means that entry stimulates technological progress.

[13]To see that $r - m_a.\hat{A}_i$ is the marginal opportunity cost of investing in productivity, note that the direct rate of return on investment is r; the increase of the productivity value is $(m_a.\hat{A}_i)$. A dollar invested in a bond will yield e^{rt} in t periods, whereas a dollar invested in productivity yield $e^{m_a.\hat{A}_i.t}$ in t periods because of appreciation. The opportunity cost is $e^{-rt} - e^{m_a.\hat{A}_i.t}$, so that the marginal opportunity cost at t=0 is $r - m_a.\hat{A}_i$. See, also Nerlove and Arrow (1962).

5 Steady State Growth Rates

The steady state growth rates are calculated in appendix B. Equation (B.26) states the steady state growth rate of process and product R&D[14]:

$$\hat{R}^{*} = M \,.\, \left[\varepsilon.\hat{X}_0 - \hat{q}_c - (\varepsilon-1).\hat{Z}_1 + (\varepsilon-1).(b-\xi).\left(\frac{\delta_2}{1-m_q}\right).\hat{S}_d + \left(\frac{\theta_2}{1-m_a}\right).\hat{S}_c \right] \qquad \textbf{(5.1)}$$

where $\hat{Z}_1 = (1-\alpha).\hat{w} + \alpha.\hat{v}$ and M is the steady state growth rate multiplier.

$$M = \frac{1}{1-(\varepsilon-1).(b-\xi).\left(\frac{\delta+\delta_1-\delta_3}{1-m_q}\right) - (\varepsilon-1).\left(\frac{\theta+\theta_1}{1-m_a}\right)} \qquad \textbf{(5.2)}$$

For expository convenience we have dropped the subscript i. This can be done because firms take the same decisions in a symmetric equilibrium. Before equation (5.1) will be discussed, a few assumptions will be made. First, the value of the multiplier, M, is positive and secondly, $b>\xi$. The first assumption is made to derive a meaningful situation in which for example the growth rate of the price of R&D has a negative influence on the growth rate of R&D[15]. The second assumption is required to make product R&D profitable; and states that the demand creating effect of an increase in quality (b) is larger than the cost increasing effect of quality (ξ).

Nested within equation (5.1) is the result derived by Van Meijl and Van Zon (1993) which is when $\xi = m_q = m_a = \theta_1 = \delta_1 = 0$ and $\hat{S}_c = \hat{S}_d = 0$. The focus of the discussion on the steady state growth rates is therefore especially on these parameters and growth rates.

The influence of the various spillover effects is quite different. The "extent" of intra-industry spillovers (ω_a or ω_q) has no influence on the steady state growth rate of R&D. The negative influence of this effect on the R&D level found by Spence (1984) and Levin and Reiss (1988), can therefore not be observed for the R&D growth rates. The "productivity" (θ_1, δ_1) of the intra-industry process and product R&D spillovers increase the value of the multiplier. They therefore strengthen the positive or negative effect of

[14]To simplify the steady state growth rate equation we made the not uncommon assumption that the growth rates of the prices of process and product R&D are equal. As a result the $\hat{q}_d - \hat{q}_c$ term in equation (B.26) cancels out. Another result of this assumption is that the growth rates of process and product R&D are equal, see equation (B.28), i.e. $R_{c,i}^* = R_{d,i}^* = R^*$.

[15]In our analyses of the dynamics of the system we also find that when this condition holds we have a saddle point stable situation and when this is not the case the system becomes unstable, see appendix D.

exogenous growth rates. The productivity of the threatening spillover pool (δ_3) decreases the value of the growth rate multiplier. The productivity and growth rate of inter-industry spillovers have a positive influence on the steady state R&D growth. Finally, it is apparent that product (process) R&D spillovers have also a positive influence on the growth rate of process (product) R&D.

The denominator of the growth rate multiplier includes two terms that describe the influence of product and process R&D. Let's start with the influence of process R&D, the third term in the denominator. A firm engages in process R&D to increase productivity. Own process R&D is productive in three ways; one direct productivity effect (θ) and two indirect productivity effects because it contributes to intra-industry R&D (θ_1) and to the own knowledge stock (m_a). The total influence of process R&D on productivity is therefore given by $(\theta+\theta_1)/(1-m_a)$. The increased productivity, induced by process R&D, decreases the price level and increases revenue by (ε-1) times the productivity change. The total third term characterises therefore the *cost-reducing effect*.

A similar explanation can be found for the second term which embodies the product R&D effect. A marginal increase in product R&D leads to a change in quality of $(\delta+\delta_1-\delta_3)/(1-m_q)$. This change in quality influences the demand direct by b.(ε-1) and indirect via higher costs $\rightarrow$ higher pricelevel by ξ $\rightarrow$ lower demand by $\xi.\varepsilon$ $\rightarrow$ lower revenue by $\xi.(\varepsilon-1)$. The second term represents therefore the *demand creating effect*.

The cost-reducing and demand creating effect increase both the value of the growth rate multiplier. The value of the multiplier is therefore higher than in a situation where one can only engage in either product R&D or process R&D. The cost reducing and demand creating effect reinforce each other in stead of compete with each other.

Finally, the growth rate of the autonomous scale of demand has a positive influence and the growth rates of unit costs and the price of process R&D have a negative influence on $\hat{R}_c^*$[16].

6 Dynamics

The dynamic behaviour of this system can be studied by analyzing the four differential equations. These turn out to be non-linear and dependent on time[17], which makes the system analytically intractable. A qualitative graphic

[16] The wage and user cost of capital growth rate have a negative influence on the R&D steady state growth rate, which seems counter intuitive because higher input prices imply that the potential benefits of process R&D per unit of output increase. But in a situation with a price-elastic demand curve (a, ε>1) higher input prices imply also that the price level will increase which decreases demand more than proportionally.

[17]The time dependency is caused by the assumption that several exogenous variables, which enter the differential equations, have a constant growth rate.

or phase-diagram analysis will therefore be used to study dynamics[18]. A condition for this analysis is that the differential equations are not dependent on time. Appendix C uses therefore first a time elimination method to make the system autonomous: the endogenous variables will be deflated with their steady state growth rates.

The two dynamic constraints in terms of the redefined or deflated variables become

$$\frac{dA''}{dt} = \eta . A''^{\,m_a} . R_c''^{\,\theta} . T_c''^{\,\theta_1} . S_c''^{\,\theta_2} - \sigma_a . A'' \quad \textbf{(6.1)}$$

$$\frac{dQ_p''}{dt} = \left(\frac{\gamma . Q_p''^{\,m_q} . R_d''^{\,\delta} . T_d''^{\,\delta_1} . S_d''^{\,\delta_2}}{W_{d,i}''^{\,\delta_3}} \right) - \sigma_q . Q_p'' \quad \textbf{(6.2)}$$

where x" = deflated value of variable x[19], σ_x is the steady state growth rate of variable x. In comparison with the dynamic constraints of the original system, stated in equation (3.2) and equation (3.6), there is a depreciation factor which is equal to the steady state growth rate in the new constraints.
The dR_c"/dt and dR_d"/dt differential equation are given by equation (C.14) and equation (C.17) respectively in appendix C.

$$\frac{dR_c''}{dt} = \phi_4 . (R_c'')^{\theta+\theta_1} . \left(\phi_5 . (R_c'')^{1-\theta-\theta_1} - \phi_6 . (Q_p'')^{(b-\xi).(\varepsilon-1)} . (A'')^{\varepsilon+m_a-2} \right) \quad \textbf{(6.3)}$$

$$\frac{dR_d''}{dt} = \phi_1 . (R_d'')^{\delta+\delta_1-\delta_3} . \left(\phi_2 . (R_d'')^{1-\delta-\delta_1+\delta_3} - \phi_3 . \frac{(A'')^{\varepsilon-1}}{(Q_{p,i}'')^{1-m_q-(b-\xi).(\varepsilon-1)}} \right) \quad \textbf{(6.4)}$$

where the ϕ_i's are described in appendix C and only dependent on a bunch of parameters and exogenous variables.

A graphical illustration of the dynamic behaviour of Q_p", A", R_d" and R_c" is impossible because this requires a four-dimensional space. Nevertheless, it is possible to divide the total system in two parts. One subsystem describes the relation between process R&D and productivity given the quality level and the other characterises the relation between product R&D and quality given the productivity level.

[18] For details of the qualitative-graphic analysis of a nonlinear differential-equation, see A.C. Chiang (1984). Concerning the value of this approach Chiang says: 'The two variable phase diagram,.., is limited in that it can only answer qualitative questions- those concerning the location and dynamic stability of the intertemporal equilibrium(s). But,.., it has the compensating advantages of being able to handle nonlinear systems as comfortable as linear ones and to address problems couched in terms of general functions as readily as those in terms of specific ones' (Chiang (1984), p 629).

[19]The mathematical expression of this deflation method is stated in equation (C.1) in appendix C.

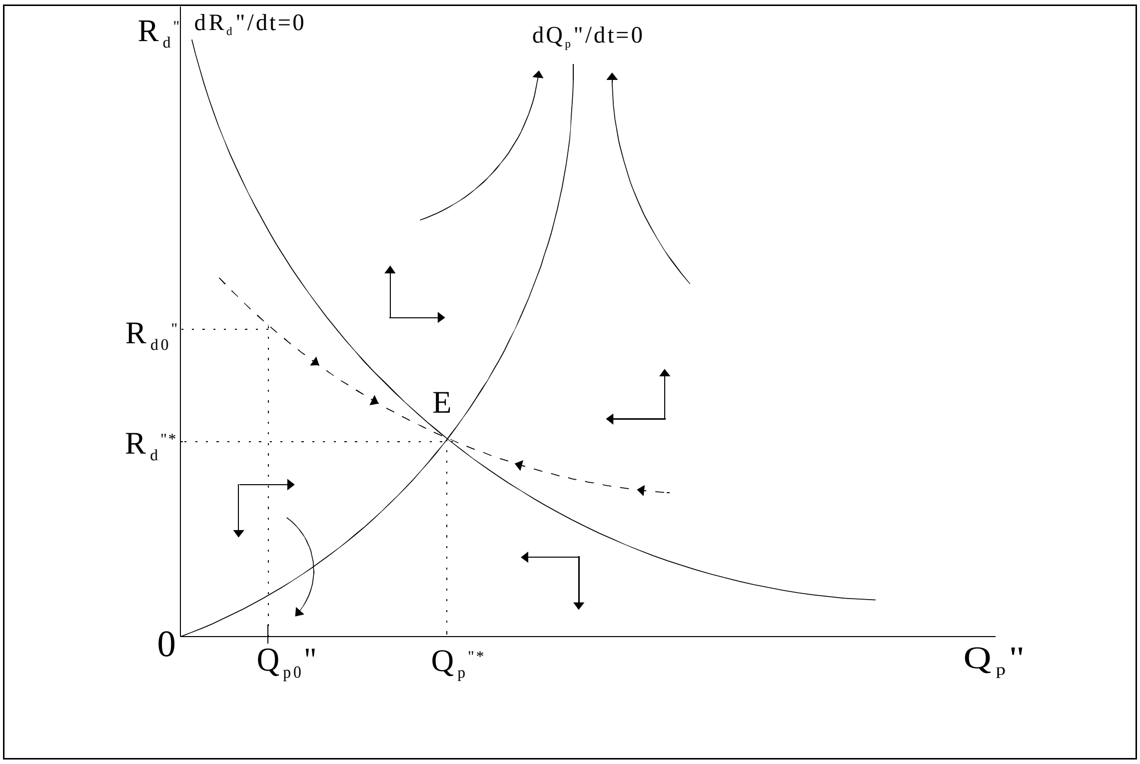

Figure 6.1, Phasediagram: Product R&D and Demand Creation

Quality Generation

Lets assume that productivity improvements are not possible. A firm therefore engages only in product R&D to enhance its quality level and takes its productivity level as given. The dynamics of this system are characterised by equation (6.2) and equation (6.4). These differential equations together describe the movement of Q_p'' and R_d'' in the phase diagram. The two demarcation lines, $dQ_p''/dt=0$ and $dR_d''/dt=0$, which describe any potential equilibrium are

$$\frac{dQ''_{p,i}}{dt} = 0 \quad \Rightarrow \quad R''_{d,i} = \left(\frac{\sigma_q.(n-1)^{\delta_3}}{\gamma.S_{d,0}^{\delta_2}.(1+\omega_q.(n-1))^{\delta_1-\delta_3}}\right)^{\frac{1}{\delta+\delta_1-\delta_3}}.(Q''_{p,i})^{\frac{1-m_q}{\delta+\delta_1-\delta_3}} \qquad \textbf{(6.5)}$$

$$\frac{dR''_{d}}{dt} = 0 \quad \Rightarrow \quad R''_{d} = \left[\left(\frac{\phi_3}{\phi_2}\right).\frac{(A'')^{\varepsilon-1}}{(Q''_{p})^{1-m_q-(b-\xi).(\varepsilon-1)}}\right]^{\frac{1}{1-\delta-\delta_1+\delta_3}} \qquad \textbf{(6.6)}$$

The $dQ_p''/dt=0$ is drawn in figure 6.1 and is positively sloped ($m_q<1$ and we assume $\delta+\delta_2-\delta_3>0$). The slope of the $dR_{d,i}''/dt=0$ locus is negative when $1>m_q+(b-\xi).(\varepsilon-1)$ which is most likely to be the case. We will first discuss this situation, which is drawn in figure 6.1. The horizontal and vertical arrows depict the movement of the system in every point in the phase diagram. The arrow configuration of figure 6.1 implies saddle point stability[20]. If the equilibrium state (i.e., that point at which both $dQ_{p,i}''/dt=0$ and $dR_{d,i}''/dt=0$) is saddle point stable, then there exist exactly one pair of trajectories of the system which lead to this equilibrium as $t\to\infty$, see Pontryagin (1962). These trajectories are illustrated in figure 6.1 by the dotted lines which lead to the equilibrium $(Q_p^{//\,*}, R_d^{//\,*})$. This unique or saddle path is also the optimal path, because all other paths ultimately lead to an infinitely large level of Q_p'' and R_d'' or to a zero level of perceived quality[21]. Given the firms' initially quality level, $Q_{p,0}''$, the optimal product R&D level to be chosen is the corresponding point on the saddle path, $R_{d,0}''$.

Lets continue the discussion about the slope of the $dR_d''/dt=0$ locus. When $1=m_q+(b-\xi).(\varepsilon-1)$ the locus is horizontal and when $1>m_q+(b-\xi).(\varepsilon-1)$ the slope of the locus becomes positive. According to the arrow configurations stays the system saddle point stable as long as the slope of the $dA''/dt=0$ locus is steeper than the slope of the $dR_d''/dt=0$ locus. The system is therefore saddle point stable as $1>m_q+(b-\xi).(\varepsilon-1).(\delta+\delta_1-\delta_3)$ and unstable as $1\leq m_q+(b-\xi).(\varepsilon-1).(\delta+\delta_1-\delta_3)$[22]. The system is therefore stable as long as the quality generating

---

[20]There exist exactly one pair of trajectories which lead to the equilibrium as $t\to\infty$ (Pontryagin (1962) p.246).

[21] The intuition for the non-optimality of an infinitely large (Q_p'', R_d'') or a zero level of perceived quality level is that if your quality level is perceived zero by consumers your expected demand is also zero, which is no economic viable situation. An infinitely large level of product R&D and perceived quality can also not be optimal because there are diminishing return to product R&D in the productivity generation process and to demand creating effects of quality. For a technical discussion of these statements see Van Meijl and Van Zon (1993).

[22]We checked the saddle point stability by a first order Taylor expansion of the non-linear differential system around its equilibrium. The Jacobian matrix evaluated at the steady-state point (E) is

$$J_E = \begin{bmatrix} -(1-m_q).\sigma_q & \dfrac{(\delta+\delta_1-\delta_3).(dQ_p^{//}/dt+\sigma_q.Q_p^{//})}{R_d^{//}} \\ (1-m_q-(b-\xi).(\varepsilon-1)).\phi_1.\phi_3.\dfrac{(R_d^{//})^{\delta-\delta_1-\delta_3}.A^{\varepsilon-1}}{Q_p^{2-m_q-(b-\xi).(\varepsilon-1)}} & \phi_1.\phi_2.(1-\delta-\delta_1+\delta_3) \end{bmatrix}$$

It is easy to verify that the determinant of the Jacobian matrix can eventually be written as

$$r_1.r_2 = |J_E| = -\phi_1.\phi_2.\sigma_q.\left[1-m_q-(b-\xi).(\varepsilon-1).(\delta+\delta_1-\delta_3)\right]$$

The value of the determinant is negative as $1>m_q+(b-\xi).(\varepsilon-1).(\delta+\delta_1-\delta_3)$. This enables us immediately to conclude that the system is saddlepoint stable because the two characteristic roots r_1 and r_2 have opposite signs. When $1\leq m_q+(b-\xi).(\varepsilon-1).(\delta+\delta_1-\delta_3)$ the determinant of the Jacobian matrix is positive and we are not able to make directly inference about the local stability of the system. To make inference we have to calculate the trace of the Jacobian matrix: tr $J_E=r_1+r_2=r-\sigma$. The trace is positive if $r>\sigma$. This condition is fulfilled if the transversality condition is satisfied (see appendix B, equation (B.13)). A positive value of both the determinant and the trace of the Jacobian matrix implies a locally unstable equilibrium. For more details of the procedure of linearization opportunities are not too large.

Lets now study what happens if the given productivity level changes. If the value of A_p'' changes we get only a shift of the $dR_d''/dt=0$ locus. This locus moves upwards (downwards) as the productivity level is higher (lower). These movements have no influence on the stability characteristics. The $dR_d''/dt=0$ locus shifts because on this locus the marginal costs of performing product R&D are equal to the marginal benefits. A higher (lower) productivity level increases (decreases) the benefits while cost stay the same. This results in an upward (downward) shift of the $dR_d''/dt=0$ locus.

Productivity Improvement

Now lets turn things around and assume that only productivity improvements are possible. The dynamics of this system are characterised by equation (6.1) and equation (6.3), which describe the movement of A'' and R_c'' in the phase diagram. The two demarcation lines are

$$\frac{dA''}{dt} = 0 \quad\Rightarrow\quad R_c'' = \left(\frac{\sigma_a}{\eta . S_{c,0}^{\theta_2}.(1+\omega_a.(n-1))^{\theta_1}}\right)^{\frac{1}{\theta+\theta_1}}.(A'')^{\frac{1-m_a}{\theta+\theta_1}} \qquad \textbf{(6.7)}$$

$$\frac{dR_c''}{dt} = 0 \quad\Rightarrow\quad R_c'' = \left(\frac{\phi_6}{\phi_5}\right)^{\frac{1}{1-\theta-\theta_1}}.(Q_p'')^{\frac{(b-\xi).(\varepsilon-1)}{1-\theta-\theta_1}}.(A'')^{\frac{\varepsilon+m_a-2}{1-\theta-\theta_1}} \qquad \textbf{(6.8)}$$

The $dA''/dt=0$ is positively sloped, $m_a<1$. The slope of the $dR_c''/dt=0$ locus is dependent on the value of the general price elasticity, ε and the value of the knowledge stock elasticity of the productivity generation process, m_a. When $\varepsilon+m_a<2$ the $dR_c''/dt=0$ locus has a negative slope. The locus is horizontal when $\varepsilon+m_a=2$ and has a positive slope as $\varepsilon+m_a>2$. This situation is analogous to the quality generation case and the phase diagram associated with this system looks similar to figure 6.1. When the price elasticity of demand gets higher the $dR_c''/dt=0$ rotates clockwise. The system stays saddle point stable as long as the slope of the $dA''/dt=0$ locus is steeper than the slope of the $dR_c''/dt=0$ locus. The system is therefore saddle point stable as long as the price elasticity

of a non-linear system, see A.C. Chiang (1984) section 18.6.

of demand is $1<\varepsilon<(1-m_a+\theta+\theta_1)/(\theta+\theta_1)$ and unstable as $\varepsilon\geq(1-m_a+\theta+\theta_1)/(\theta+\theta_1)$[23].

Quality and Productivity Generation

By integrating the two sub-systems the dynamics can be examined of the situation in which both quality and productivity changes are possible. It will be assumed that the initial quality and productivity levels are lower than their steady state levels ($Q_{p,0}'' < Q_p''^{*}$, $A_0'' < A''^{*}$). Furthermore we assume that $\varepsilon<2-m_a$ and $1>m_q+(b-\xi).(\varepsilon-1)$ which implies that both sub- systems are saddle point stable and that the $dR_c''/dt=0$ and $dR_d''/dt=0$ loci have both a negative slope.

In phase diagram 6.2 the demarcation loci of the quality generating sub-system are drawn. The $dQ_p''/dt=0$ locus is independent and the $dR_d''/dt=0$ locus is dependent on the productivity level. At time t=0 is the $(dR_d''/dt)_0=0$ locus drawn dependent on the initially productivity level, A_0''. The steady state is located in point E_0 and the associated saddlepath is s_0. Given its initial quality level, $Q_{p,0}''$, it is optimal for the firm to choose the product R&D level of $R_{d,0}''$ on this saddle path.

[23]In a similar way as with the quality generation system we checked the saddle point stability by a first order Taylor expansion of the non-linear differential system around its equilibrium. The Jacobian matrix evaluated at the steady-state point (E) is

$$J_E = \begin{bmatrix} -(1-m_a).\sigma_a & \dfrac{(\theta+\theta_1).dA''/dt}{R_c''} \\ -(\varepsilon+m_a-2).\phi_4.\phi_6.(R_c'')^{\theta-\theta_1}.\dfrac{(A'')^{\varepsilon-m_a-3}}{Q_p''^{(b-\xi).(\varepsilon-1)}} & \phi_4.\phi_5.(1-\theta-\theta_1) \end{bmatrix}$$

It is easy to verify that the determinant of the Jacobian matrix can eventually be written as

$$r_1.r_2 = |J_E| = -\phi_4.\phi_5.\sigma_a.\left[1-m_a-(\varepsilon-1).(\theta+\theta_1)\right]$$

The value of the determinant is negative as $1>m_a+(\varepsilon-1).(\theta+\theta_1)$, which implies saddle point stability. When $1\leq m_a+(\varepsilon-1).(\theta+\theta_1)$ the determinant is positive. The value of the trace is again r-σ which implies together with the positive determinant a locally unstable equilibrium.

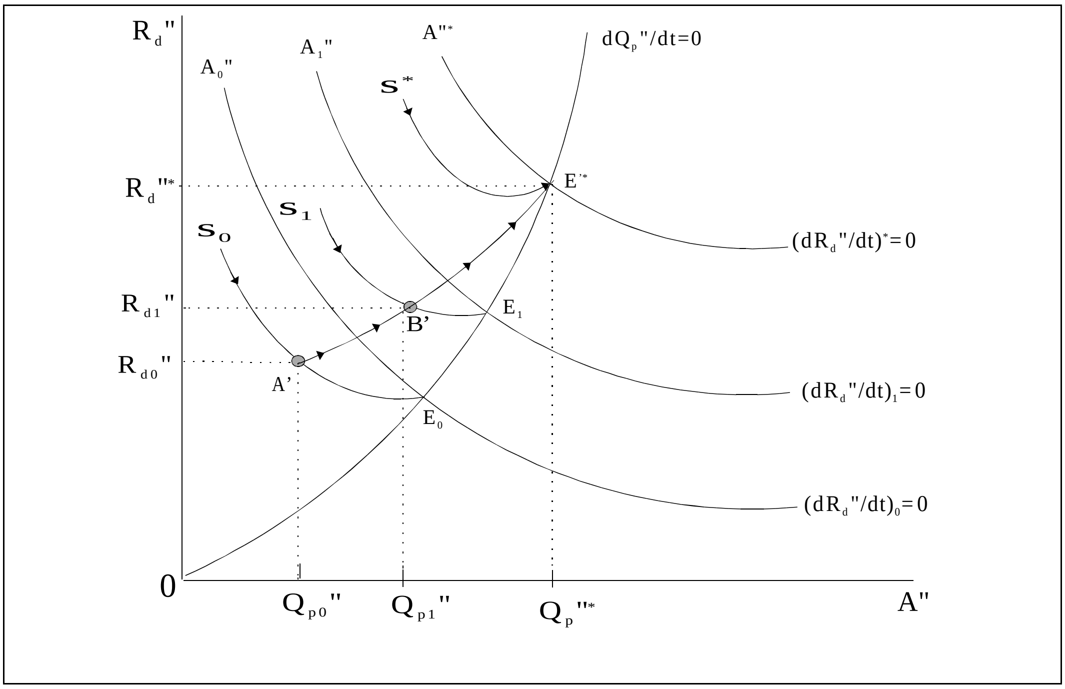

Figure 6.2, Phasediagram: Subsystem: Product R&D and Demand Creation

In phasediagram 6.3 the demarcation loci of the productivity generating sub-system are drawn. The $(dR_c"/dt)_0$ locus is drawn dependent on the initially given quality level, $Q_{p,0}$". The steady state is located in point E_0' and the accompanying saddle path is s_0'. A firm with an initially productivity level of A_0" will choose the process R&D level of $R_{c,0}$".

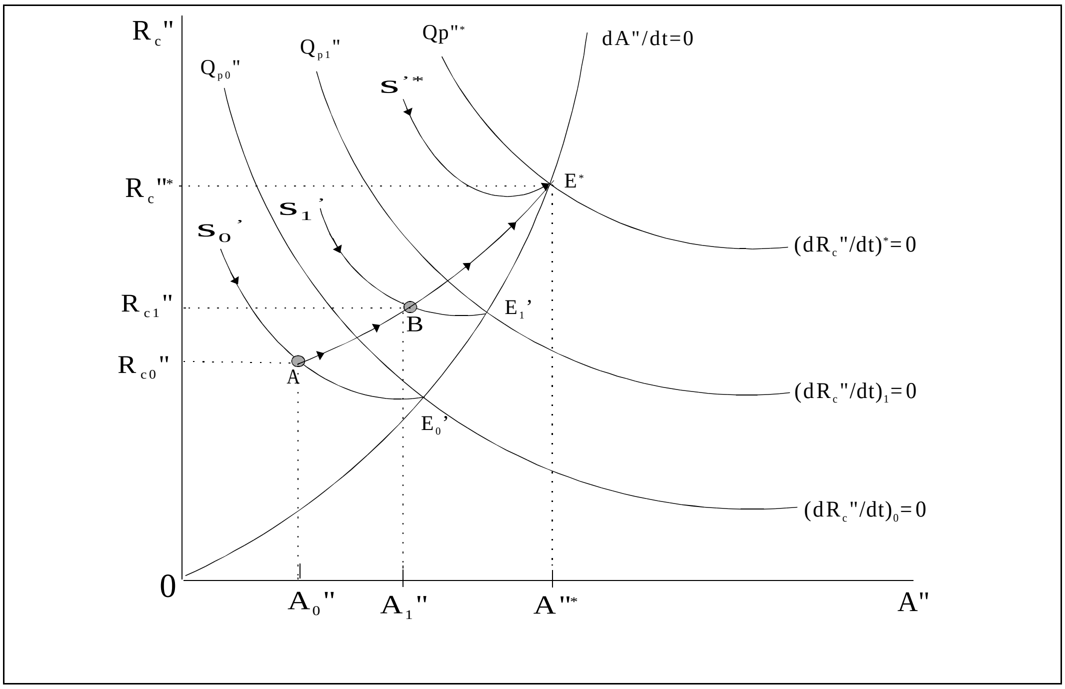

Figure 4.3, Phasediagram: Subsystem: Process R&D and Productivity

Given the firms' choice of the quantity of product and process R&D level at time t=0 the values of its new productivity level and perceived quality level are (approximately):

$$A_1'' = A_0'' + \eta . A_0''^{\,m_a} . R_{c,0}''^{\,\theta} . T_{c,0}''^{\,\theta_1} . S_{c,0}''^{\,\theta_2} - \sigma_a . A_0'' \quad \mathbf{(6.9)}$$

$$Q_{p,1}'' = Q_{p,0}'' + \left(\frac{\gamma . Q_{p,0}''^{\,m_q} . R_{d,0}''^{\,\delta} . T_{d,0}''^{\,\delta_1} . S_{d,0}''^{\,\delta_2}}{W_{d,i}''^{\,\delta_3}} \right) - \sigma_q . Q_{p,0}'' \quad \mathbf{(6.10)}$$

The new productivity level, A_1", and the new perceived quality level, $Q_{p,1}$", have both increased. A higher productivity level, A_1", shifts the dR_d"/dt=0 locus upwards to $(dR_d"/dt)_1=0$ in figure 6.2. The steady state equilibrium shifts from E_0 to E_1. A higher productivity level implies a higher steady state perceived quality level. The higher perceived quality level, $Q_{p,1}$", on the other hand, shifts the dR_c"/dt=0 locus upwards to $(dR_c"/dt)_1=0$ in figure 6.3. The steady state equilibrium shifts from E_0' to E_1'. A higher perceived quality level leads therefore to a higher steady state productivity level. This analysis shows that process and product R&D reinforce eachother instead of competing with each other, or are complements in stead of substitutes.

This process continues until the steady state values, $Q_p"^*$ and $A"^*$, are approached. When the productivity level and perceived quality level of the firm are equal to the steady state values, the firm will choose the process and product R&D levels which are just enough to maintain the current perceived quality and productivity levels. The dR_c"/dt=0 and dR_d"/dt=0 will not shift anymore and the firm will choose $R_d"^*$ and $R_c"^*$ from now on. We have reached a steady state in which the discounted values of Q_p and A are constant and $Q_{p,t}$ and A_t grow at a constant rate. When the initial productivity and perceived quality level are not equal to these steady state values we get a long-run adjustment path for the amount of process R&D and product R&D which is given by respectively A'B'E'* in figure 6.2 and ABE* in figure 6.3.

We used the three conditions of Feichtinger and Hartl (1986) to examine the validity of the saddle point steady state property of this four dimensional differential equation system[24]. The system is saddle point stable when two characteristic roots are real and negative and the other two characteristic roots are real and positive. In that case the system possesses a saddle point plane. When we know the two initial conditions A_0" and $Q_{p,0}$", the begin point of the unique dynamic path in this saddlepoint plane is exactly determined. We checked in appendix D the conditions of Feichtinger and Hartl for a saddle point plane and concluded that this system is saddle point stable when holds:

[24]For details on the stability of a non-linear differential system with more than one state variable see Feichtinger and Hartl (1986) p. 122-154.

$$1>(b-\xi).(\varepsilon-1).\left(\frac{\delta+\delta_1-\delta_3}{1-m_q}\right)+(\varepsilon-1).\left(\frac{\theta+\theta_1}{1-m_a}\right) \qquad \textbf{(6.11)}$$

This expression is familiar to us because it is the denominator of the growth rate multiplier, see equation (5.2).

7 Steady State

The steady state values of the redefined system can be obtained by putting the four differential equations equal to zero. Appendix C shows these calculations.

7.1 R&D Intensities

An important measure of technological change is the R&D intensity of production. The R&D intensity is defined as the R&D expenditures to total revenue ratio. In this paragraph we investigate which factors favour or temper this technological change measure. The steady state product and process R&D intensities are given by equation (C.25) and equation (C.26) respectively.

$$\phi_c = \left(1-\frac{1}{a_p}\right).\left[\theta+\frac{\theta_1}{1+\omega_a.(n-1)}\right].\left(\frac{\sigma_a}{r-\sigma+(1-m_a).\sigma_a}\right) \qquad \textbf{(7.1)}$$

$$\phi_d = (b-\xi).\left(1-\frac{1}{a_p}\right).\left(\delta+\frac{\delta_1-\omega_q.\delta_3}{1+\omega_q.(n-1)}\right).\left(\frac{\sigma_q}{r-\sigma+(1-m_q).\sigma_q}\right) \qquad \textbf{(7.2)}$$

where ϕ_c denotes the process R&D intensity and ϕ_d denotes the product R&D intensity.

It is immediately apparent that the perceived price elasticity, a_p=a-(a-ε)/n, has a positive influence on both R&D intensities. For a given number of firms, signifies this the interesting result that the R&D intensity is higher for products which are characterised by a higher elasticity of substitution (a) or a higher general price elasticity (ε). The explanation for this is that a higher perceived price elasticity, which implies also a higher perceived quality elasticity, means that productivity and quality improvements are expected to result in larger changes in demand.

A higher elasticity of substitution means that the "Love of Variety" effect is lower (see section 2). Variety per se, is valued less by consumers which implies that the products are perceived as less differentiated by consumers and that the competition between different varieties increases. We will use the elasticity of substitution as an indicator of the degree of product differentiation.

The larger the elasticity of substitution the less the degree of product differentiation. This implies that products which have a higher degree of product differentiation have a lower process and product R&D intensity.

However, the "correct" price elasticity for a firm in a symmetric industry is the interindustry price elasticity, ε, which is smaller than the perceived elasticity. The assumption of a given price and quality level of competitors, which is the cause of the higher perceived price elasticity, therefore favours technological change. If the firms had taken into account the reactions of other firms, they faced ε as price elasticity and they would have invested less in R&D. Without spill-over effects, price setting firms therefore overinvest in process and product R&D with respect to their optimal profit level[25].

With spill-over effects there is an underinvestment in process R&D. Firms determine their optimal process R&D level while taking the process R&D level of competitors constant. The elasticity on the industry pool of process R&D is perceived as $\theta_1/(1+\omega_a(n-1))$ but is θ_1 in a symmetric industry. The perceived elasticity on the industry pool of R&D is therefore lower than the "correct" elasticity, which leads to an underinvestment in process R&D with respect to optimal profits.

With regard to the product R&D we obtain also an underinvestment in R&D if we consider only the influence of the intra industry spillover pool, $T_{d,i}$, because the elasticity of this industry pool is perceived as $\delta_1/(1+\omega_q(n-1))$ while it is δ_1 in a symmetric industry. However the perceived elasticity of the threatening intra industry spillover pool, $W_{d,i}$, is $-\delta_3.\omega_q/(1+\omega_q(n-1))$ but $-\delta_3$ in a symmetric equilibrium. The negative influence of the threatening pool is therefore underestimated which causes an overinvestment in product R&D. The total effect of both the intra-industry spillover pools on product R&D is therefore ambiguous.

The perceived price elasticity implies an overinvestment, the productivity spillover effect implies an underinvestment in the R&D intensity and the extend of the quality spillover effect is indeterminate. The net effect is ambiguous and dependent on the strength of the three effects.

Let's consider now the effects of changes in the parameters on the R&D intensities. Differentiating ϕ_c with respect to the number of firms, n, we find[26]

$$\frac{\partial \phi_c}{\partial n} = \phi_c . \left(\frac{-\omega_a . \theta_1}{\theta.(1+\omega_a.(n-1))^2 + \theta_1.(1+\omega_a.(n-1))} + \frac{a-\varepsilon}{(n.(a-1)-a+\varepsilon).(n.a-a+\varepsilon)} \right) \qquad \textbf{(7.3)}$$

The first term between the brackets has a negative value and represents the disincentive effect of entry when a part of the firms knowledge can be used by other firms and it can also use knowledge of other firms. It can use R&D of other firms without costs. This is therefore a disincentive for the R&D intensity. The larger the "extent" of the spillover, ω_a, the larger this disincentive

[25]See, also Delbono and Denicolo (1990).

[26]Remember that a_p is also dependent on n: a_p=a-(a-ε)/n.

effect. But on the other hand a larger number of firms implies a higher perceived price elasticity, which increases the perceived change in demand of a productivity improvement. This effect, which is represented by the second term between the brackets favours the R&D intensity. The value of the second term is therefore positive. This effect increases with the value of the intra-industry price elasticity.

The net effect of an increase in the number of firms on the R&D elasticity is dependent on the strength of the two opposite effects. The R&D intensity decreases with n if and only if the sum of the two terms between the brackets is negative. This is more likely to be the case when ω_a is large or the difference between a and ε is small. Figure 7.1 and figure 7.2 illustrate the essential tendencies. All the figures in this section are obtained from simulations which are executed with the values of the parameters and exogenous variables described in appendix E. Only if different values for some parameters are used then those described in appendix E we report their values.

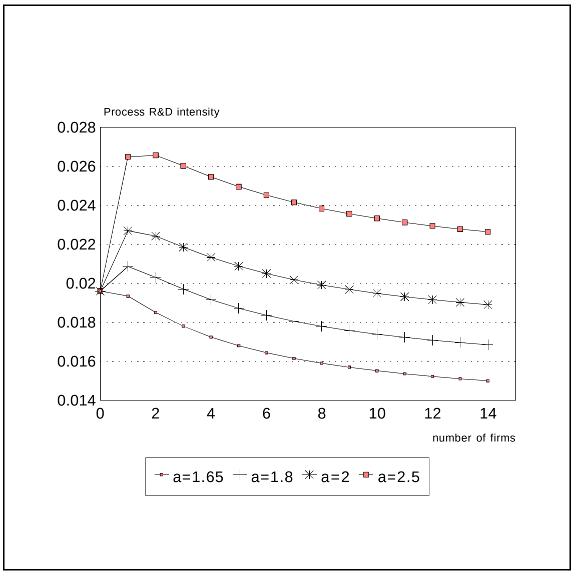

Figure 7.1: The influence of entry on the process R&D intensity given various degrees of product differentiation and ω_a=0.5.

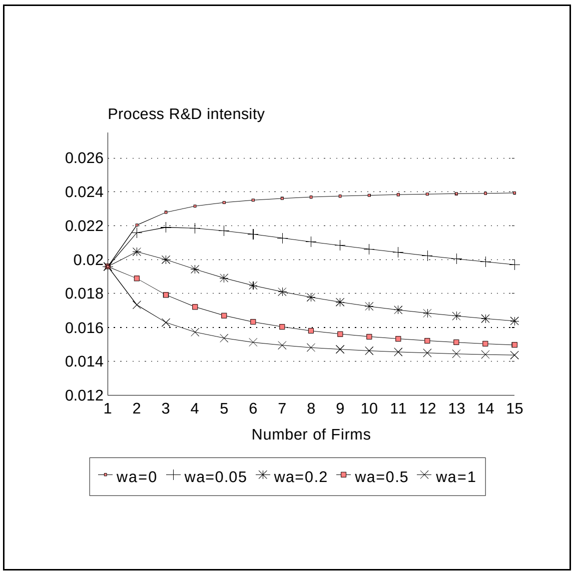

Figure 7.2: The influence of entry on the process R&D intensity given different spillover levels and a=1.7. (wa in the legend represents ω_a)

Figure 7.1 shows that given the inter industry price elastici ε moderate extent of spillovers, ω_a=0.5, a higher elasticity of substitution, a, causes the R&D intensity to rise. When the elasticity of substitution is low the R&D intensity decreases with entry. With a higher intra industry price elasticity the R&D intensity first increases and then decreases with entry.

Figure 7.2 shows the influence of entry on the R&D intensity for various levels of the extent of spillovers (ω_a). When a larger part of foreign R&D can be used, higher value of ω_a, the R&D intensity decreases. When spillovers are not present, ω_a=0, then the R&D intensity increases with entry. If the spillovers are perfect, ω_a=1, the R&D intensity decreases with entry. For levels in between the R&D intensity first increases and then decreases.

The derivatives of the R&D intensity with respect to the other parameters are given in the table below

	Derivative of ϕ_c	Derivative of ϕ_d
θ	$\phi_c/(\theta+\theta_1/(1+\omega_a(n-1)) > 0$	0
θ_1	$\phi_c/(\theta.(1+\omega_a(n-1)+\theta_1) > 0$	0
δ	0	$\phi_d/(\delta+(\delta_1-\omega_q.\delta_3)/(1+\omega_q(n-1)) > 0$
δ_1	0	$\phi_d/(\delta.(1+\omega_q(n-1)+\delta_1-\omega_q.\delta_3) > 0$
δ_3	0	$-\phi_d.\omega_q/(\delta.(1+\omega_q(n-1)+\delta_1-\omega_q.\delta_3) < 0$
ω_a	$\frac{-\theta_1.(n-1).\phi_c}{\theta.(1+\omega_a.(n-1))^2+\theta_1.(1+\omega_a(n-1))} < 0$	0
ω_q		$\frac{(-\delta_1.(n-1)-\delta_3).\phi_d}{\delta.(1+\omega_q.(n-1))^2+(\delta_1-\omega_q.\delta_3).(1+\omega_q(n-1))} < 0$
n	$\frac{-\omega_a.\theta_1.\phi_c}{\theta.(1+\omega_a.(n-1))^2+\theta_1.(1+\omega_a(n-1))} + \frac{a-e}{(n.(a-1)-(a+e)).(n.a-a+e)} \lessgtr 0$	$\frac{-\omega_q.(\delta_1-\delta_3.\omega_q).\phi_d}{\delta.(1+\omega_q.(n-1))^2+(\delta_1-\omega_q.\delta_3).(1+\omega_q(n-1))} + \frac{a-e}{.(b-\xi).(n.(a-1)-a+e)).(n.a-a+e)} \lessgtr 0$
b	0	$\phi_d/(b-\xi) > 0$
ξ	0	$-\phi_d/(b-\xi) < 0$
a	$\phi_c.n.(n-1)/((n.(a-1)-a+e).(n.a-a+e)) > 0$	$\phi_d.n.(n-1)/((n.(a-1)-a+e).(n.a-a+e)) > 0$
ε	$\phi_c.n/((n.(a-1)-a+e).(n.a-a+e)) > 0$	$\phi_d.n/((n.(a-1)-a+e).(n.a-a+e)) > 0$
σ_a	$\frac{(r-\sigma).\phi_c}{\sigma_a.(r-\sigma+(1-m_a).\sigma_a)} > 0$	0
σ_q	0	$\frac{(r-\sigma).\phi_d}{\sigma_q.(r-\sigma+(1-m_q).\sigma_q)} > 0$
r	$-\phi_c\ /(r-\sigma+(1-m_a).\sigma_a) < 0$	$-\phi_d\ /(r-\sigma+(1-m_q).\sigma_q) < 0$
σ	$\phi_c\ /(r-\sigma+(1-m_a).(w_a+\sigma_a)) > 0$	$\phi_d\ /(r-\sigma+(1-m_q).\sigma_q) > 0$

Larger technological opportunities represented by θ, θ_1, δ, δ_1 and b increase the process of product R&D intensity. The productivity of the threatening spill over pool has a negative influence on the product R&D intensity. Cost increasing effects such as the discount rate, r, and quality elasticity of costs, ξ, decrease the R&D intensity just as the extent of spillover effect. Higher steady state growth rates, $\sigma/\sigma_q/\sigma_a$ increase the R&D intensity. These results are not dependent on the "double counting" method which can easily be checked by setting θ and δ equal to zero.

7.2 R&D Levels and Spillover Effects

In this subsection we will investigate the relation between the steady state process and product R&D level and the extent of process and product R&D spillovers. The steady state levels of process and product R&D are calculated in appendix C and stated in respectively equation (C.18) and equation (C.22).

Let's first consider the case of process R&D spillovers. In general it can be expected that larger spillovers tend to reduce cost reducing innovations (Spence (1984)). De Bondt *et al* (1992), page 41-43, state that this tendency may be reversed in moderately to highly differentiated oligopolies with low R&D costs (high technological opportunities) and that this is more likely to happen if leakages and the number of firms are not too high. Their argument is that "the benefits of cost reduction can be better appropriated in these circumstances and high technological opportunities make sure increases in investments are not very costly".

Differentiating $R_c''^*$ with respect to the extent to which knowledge spillovers to rival firms, ω_a, we find

$$sign\left(\frac{\partial R_c^{//\,*}}{\partial\omega_a}\right) = sign\left[\frac{\varepsilon-1}{1-m_a}.(1+\omega_a.(n-1)) - \frac{1-(b-\xi).(\varepsilon-1).\left(\dfrac{\delta+\delta_1-\delta_3}{1-m_q}\right)}{\theta+\dfrac{\theta_1}{1+\omega_a.(n-1)}}\right] \qquad \textbf{(7.4)}$$

The net effect of ω_a on the process R&D level is ambiguous and dependent on two effects. First, a larger extent of spillovers increases the industry pool of knowledge. This raises the productivity of own R&D which result in higher marginal benefits of R&D. This positive effect is represented by the first term between the brackets. Second, a larger extent of spillovers means that the appropriability of own R&D declines. The elasticity of process R&D, $\theta+\theta_1/(1+\omega_q.(n-1))$ decreases when ω_a increases. This negative effect is shown by the second term between the brackets.

Both effects are stronger the larger is the number of firms and the higher is the extent of spillovers. The first effect is stronger because a higher ω_a and n imply a larger knowledge stock and the second effect is stronger because these two factors decrease the appropriability of own R&D. Multiplying both expressions with $\theta+\theta_1/(1+\omega_a.(n-1))$ and using the definition of the growth rate multiplier (equation (5.2)) we obtain:

$$sign\left(\frac{\partial R_c^{//\,*}}{\partial\omega_a}\right) = sign\left[-\frac{1}{M} + \frac{\varepsilon-1}{1-m_a}.\theta.\omega_a.(n-1)\right] \qquad \textbf{(7.5)}$$

Condition (6.11) requires that M is positive. Because M is independent of n and ω_a it is clear that more rivalry and less appropriability make it more likely that the influence of ω_a on $R_c''^*$ is positive. This result is just opposite to the findings of De Bondt *et al* which is not surprising because they don't take into account the first effect: a higher knowledge stock increases the productivity of your own R&D (see, the specification of their technological generation process on page 5). Figure 7.3 illustrates the case when the number of firms is small and figure

7.4 depicts the case when the number of firms is larger.

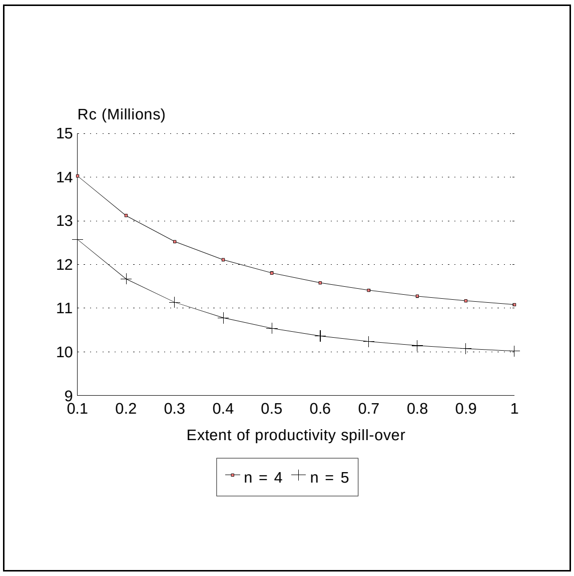

Figure 7.3: The influence of the extend of the productivity spillover on the level of process R&D, given a few firms.

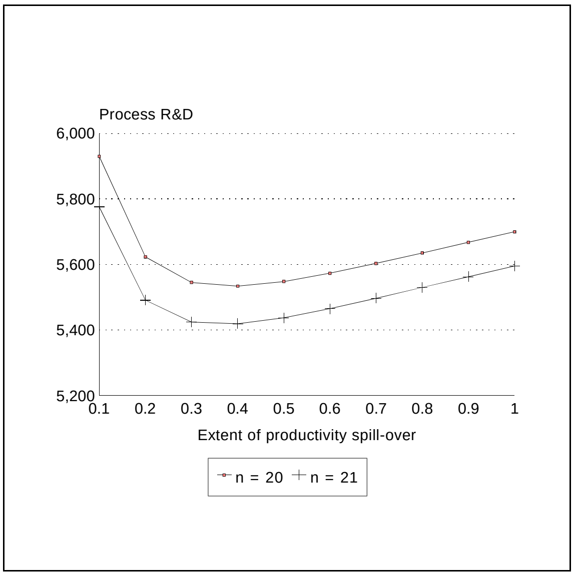

Figure 7.4: The influence of the extend of the productivity spillover level on the level of process R&D, with many firms.

Equation (7.5) shows directly that with the Spence (1984) specification, where θ=0, the second term on the right hand side vanishes and the influence of the extent of spillovers on the process R&D level is always negative. The parameter θ is therefore of crucial importance to obtain the possible positive effect of ω_a on $R_{c,i}$". The "double counting" method of Levin and Reiss causes therefore the possible positive influence.

Another important determinant of the sign of the derivative is the inter-industry or general price elasticity of demand, ε. The value of this inter-industry price elasticity can be seen as an indicator for the degree of product differentiation relative to the outside or homogenous good (H). A higher level of ε or less differentiated products increases the possibility of a positive influence of ω_a on $R_c"^*$. This effect differs also from the findings of De Bondt *et al* who conclude that the positive effect is more likely when product differentiation is moderate to high. Moderate to high product differentiation implies in their model that appropriability is larger, but in this model higher product differentiation leads to a lower perceived price elasticity of demand.

Our analysis confirms the result of De Bondt *et al* that high technological opportunities, high θ and θ_1 will make a positive influence of ω_a on $R_c"^*$ more likely. The influence of the extent of product R&D spillovers on the steady state process R&D level is again ambiguous.

$$sign\left(\frac{\partial R_c^{//\ *}}{\partial \omega_q}\right) = sign\left[(\delta_1-\delta_3).(n-1) - \frac{(\delta+\delta_1-\delta_3).(\delta_3+(n-1).\delta_1)}{\delta.(1+\omega_q.(n-1))+\delta_1-\delta_3.\omega_q}\right] \gtrless 0 \qquad \textbf{(7.6)}$$

When the productivity of the threatening spillover pool is zero (δ_3=0) then is the influence of ω_q on $R_{c,i}$ is positive. This is also always the case if one takes the Levin and Reiss specification of the threatening knowledge pool (δ_3=0 and δ_1 can be positive or negative). If there is no "double counting" (θ=0) then is the influence of ω_q on $R_{c,i}$ negative if $\delta_1>\delta_3$ and positive if $\delta_1<\delta_3$. If there is no "double counting" and in addition δ_3 is equal to zero then there is no influence of ω_q on $R_{c,i}$.

The influence of the extent of product R&D spillovers on the steady state product R&D level is again ambiguous. The possibility of a positive influence is higher the higher is the productivity of the intra industry knowledge pool $T_{d,i}$ (i.e. δ_1) and the lower is the productivity of the threatening pool $W_{d,i}$ (i.e. δ_3). Interesting is that the influence of the extent of process spillovers, ω_a, has always a positive influence on the steady state product R&D level.

7.3 Cost Reducing and Demand Creating Innovations and Spillovers

The technological performance of a firm can be measured by its steady state productivity and quality level. These are stated in respectively equation (C.23) and equation (C.24). In this subsection we investigate the relation between the technological performance and the extent of spillovers. The sign of the derivation of the steady state quality and productivity level with respect to ω_q is given by equation (7.6). The remarks which we made with regard to the influence of ω_q on $R_{c,i}$" are also valid in these cases. The influence of ω_q on $A_{c,i}$" and $Q_{p,i}$" is again ambiguous.
The influence of the extent of productivity spillovers (ω_a) on the steady state productivity is

$$sign\left(\frac{\partial A_i^{//\,*}}{\partial \omega_a}\right) = sign\left(\frac{\partial Q_p^{//\,*}}{\partial \omega_a}\right) = sign\left[1 - \frac{\theta+\theta_1}{\theta.(1+\omega_a.(n-1))+\theta_1}\right] > 0 \qquad \textbf{(7.7)}$$

In contrast to the influence of ω_q, the influence of ω_a on $A_{c,i}$" and $Q_{p,i}$" is always positive.

In conclusion we can say that the extent of productivity spillovers stimulates the innovative performance of a firm and that the extent of quality spillovers tempers or stimulates innovative performance.

7.4 Cost Reducing and Demand Creating Innovations and Entry

In this section we investigate the influence of entry on technological change in the steady state by investigating the influence of entry on the steady state productivity and quality level.

We first consider the influence of entry on the productivity level. De Bondt *et al* (1992) find that, in a homogeneous oligopoly, entry reduces the innovative output. In a differentiated industry the effect is dependent on the level of spillovers. Low spillovers decrease, moderate spillovers increase and high spillovers first increase and then decrease innovative output with entry.

The derivative of the steady state productivity level with respect to the number of firms is

$$sign\left(\frac{\partial A^{//*}}{\partial n}\right) = - \frac{(a-\varepsilon).(1-m_q)}{n.(a-1)} - \frac{\delta_3.(b-\xi).(\varepsilon-1)}{n-1}$$
$$+ \frac{(1-m_q).\varepsilon.(a-\varepsilon)}{(n.(a-1)-a+\varepsilon).(a.n-a+\varepsilon)}$$
$$+ \omega_a.\theta_1.\frac{(1-m_q-(b-\xi).(\varepsilon-1).(\delta+\delta_1-\delta_3))}{1+\omega_a.(n-1)}.\left[\frac{1}{\theta+\theta_1}-\frac{1}{\theta.(1+\omega_a.(n-1))+\theta_1}\right]$$
$$+ \frac{\omega_q.(b-\xi).(\varepsilon-1)}{1+\omega_q.(n-1)}.\left[\delta_1-\delta_3-\frac{(\delta+\delta_1-\delta_3).(\delta_1-\delta_3.\omega_q)}{\delta.(1+\omega_q.(n-1))+\delta_1-\delta_3.\omega_q}\right] \quad \textbf{(7.8)}$$

The first term states that more rivals reduce market shares which makes strategic investments in R&D less profitable. The second term implies that more rivals will cause a larger threat to your perceived quality level which discourages demand creating innovations. Section 6 showed that demand creating and cost reducing innovations reinforce eachother so that this also discourages cost reducing innovations. The third term depicts that more rivals cause a higher perceived price elasticity which encourages R&D investments. The fourth term describes that more rivals increase the intra industry process R&D spillover stock which stimulates R&D investments. The fifth term describes the effect of entry on the product R&D spillover pool which can be positive or negative. The net effect of these five effects is ambiguous.

A few simulations can illuminate the complex interplay between market structure and technological change in our model. We will discuss two important determinants of the net effect. First, the difference between the elasticity of substitution and the inter-industry price elasticity, a-ε. This effect is dependent on the perceived differentiation of a product on the intra- and inter-industry level. Secondly, the extent of the process R&D spillovers. The influence of the first determinant on the relation between entry and the productivity level is illustrated in figure 7.5 and figure 7.6 given respectively a low and a high level of the extent of productivity spillovers. The value of ε=1.5 in all the simulations. We can identify four patterns. 1) When the difference between a and ε is equal to zero or small, entry may encourage the innovative efforts independent of the spillover level and degree of product differentiation. 2) If the difference between a and ε is not too large and the extent of spillovers are low or moderate, a monopolistic situation yields the highest productivity level. With high spillovers a duopoly level of rivalry obtains

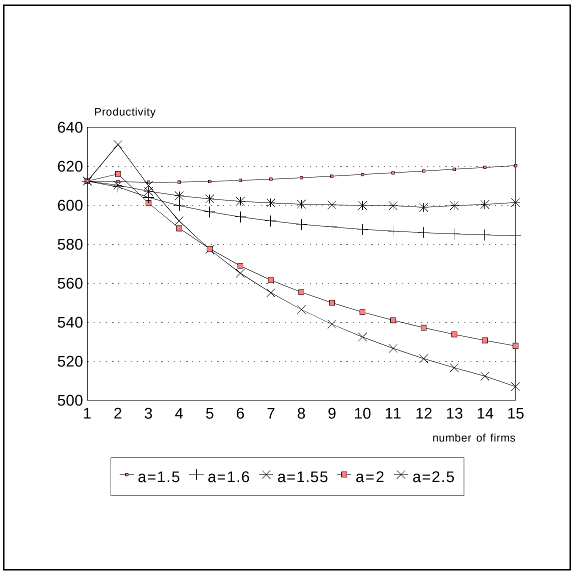

Figure 7.5: The influence of entry on the productivity level given various degrees of product differentiation and a low level of productivity spillovers (ω_a=0.1).

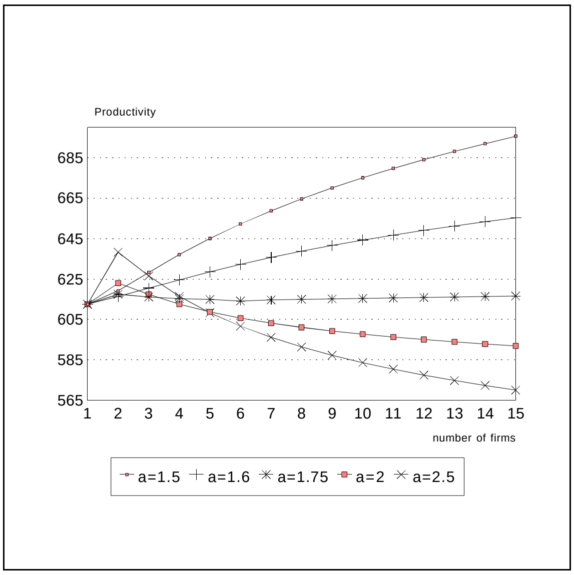

Figure 7.6: The influence of entry on the productivity level given various degrees of product differentiation and a high level of productivity spillovers (ω_a=1).

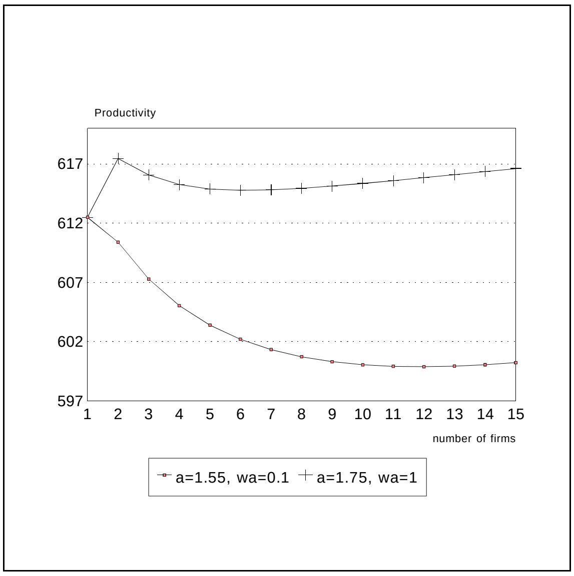

Figure 7.7: The influence of entry on productivity, given a small difference between a and ε for a high and a low level of spillovers.

the highest productivity level. 3) If the difference between a and ε is large a duopoly level of rivalry is the best independent of the level of spillovers and product differentiation. The fourth pattern, a=1.55 in figure 7.5 and a=1.75 in figure 7.6, is an interesting one and to my knowledge a new pattern to the literature. To illuminate this pattern we depicted it, given low and high spillovers, once more in figure 7.7. The productivity level first decreases and then increases with entry in the case of low spillovers. In the case of high spillovers the productivity level increases from monopoly to duopoly, then decreases until it reaches a minimum value for an intermediate level of rivalry and entry after this point increases the productivity level again. The intuition behind these results is that first the first effect (lower marketshares) drives the productivity level down. This negative effect decreases when the number of firms increases and at a certain intermediate level of rivalry become the fourth and the fifth effect (positive influence of a larger intraindustry knowledge stock) more important.

These results differs from the findings of De Bondt *et al* in the first place by the newly observed pattern and furthermore that low and moderate spillovers can decrease, increase or first increase and then decrease the innovative output with entry. Another difference is that even with high spillovers the innovative output may increase with entry.

7.5 Welfare, Entry and Spillovers

Welfare, W, is the sum of consumer surplus, CS, and total profits.

$$W = CS + n.\pi \qquad \textbf{(7.9)}$$

The consumer surplus can be defined as the utility derived from the differentiated good less the total expenditures.

$$CS = U(D) - n.p.y \qquad \textbf{(7.10)}$$

The utility function is given by equation (2.1). Using equations (2.1), (2.2), (2.6) and the first order condition (C.6) it is easy to show that

$$CS = TC^{//}.\left[1-\frac{1}{a_p}\right].\frac{1}{\varepsilon-1} \qquad \textbf{(7.11)}$$

where

$$TC^{//} = n^{\frac{\varepsilon-1}{a-1}}.X^{\varepsilon}_{o,o}.(Q^{//}_p)^{(b-\xi).(\varepsilon-1)}.(A^{//})^{\varepsilon-1}.Z^{1-\varepsilon}_o.\left[1-\frac{1}{a_p}\right]^{\varepsilon} \qquad \textbf{(7.12)}$$

The consumer surplus is positively dependent on the technological performance of firms. A higher quality and productivity level increase the consumer surplus. The direct effect of the number of firms on consumer surplus is also positive which reflect the love of variety. The love of variety is large when products are perceived very differentiated, low a, and small when productdifferentiation is not so important, high a. Finally a higher perceived price elasticity has also a positive influence on CS because this decreases the mark-up from prices over costs.

Profits, π, are defined as revenue less total costs.

$$\pi = p'' . y'' - TC'' - q_{c,o}.R_c'' - q_{d,o}.R_d'' \qquad \textbf{(7.13)}$$

Using the first order conditions (C.6)-(C.10) in combination with equation (2.6) it is straightforward to find that

$$n.\pi = TC'' . \left[\frac{1}{a_p - 1} - \frac{\sigma_a . \left(\theta + \frac{\theta_1}{1 + \omega_a.(n-1)} \right)}{r - \sigma + (1 - m_a).\sigma_a} - (b - \xi). \frac{\sigma_q . \left(\delta + \frac{\delta_1 - \delta_3.\omega_q}{1 + \omega_q.(n-1)} \right)}{r - \sigma + (1 - m_q).\sigma_q} \right] \qquad \textbf{(7.14)}$$

where n.π are total profits. Note that total profits are just as consumer surplus positively dependent on technological performance, the quality and productivity level. The term between the brackets is the net profit to total *variable* cost ratio, or for short net profit ratio. The first term between the brackets represents the gross profit ratio and the second and third term between the brackets represent respectively the process and product R&D fixed costs ratio's. The net profit ratio can be positive or negative dependent on the strength of the three effects. Hence, a negative net profit ratio implies also negative profits which is no viable situation in the long run. We will first investigate which factors make an unviable situation more likely.

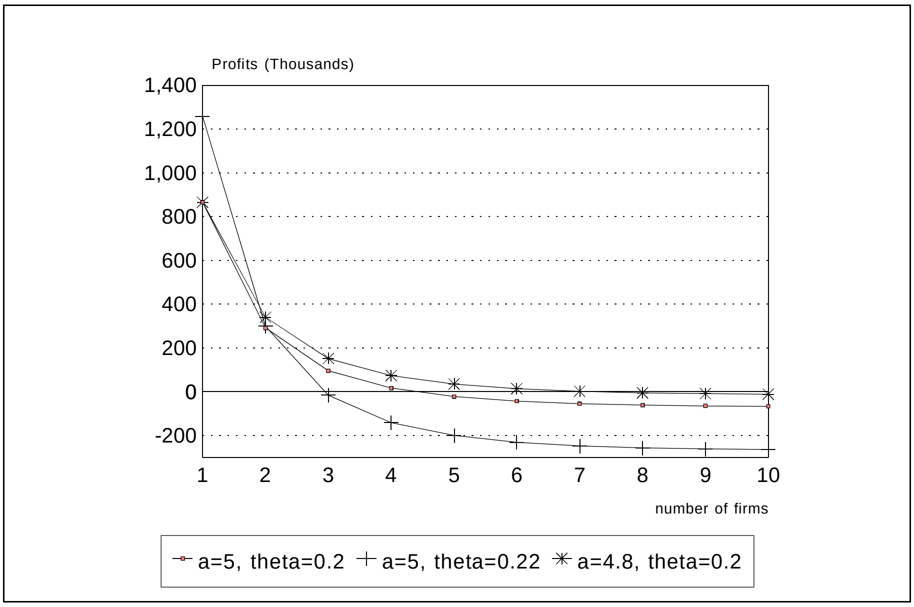

Figure 7.8: The Influence of higher technological opportunities (a higher value of θ) and a lower degree of product differentiation (a higher level of a) on the number of viable firms in an industry.

Entry increases the perceived price elasticity, which lowers the gross profit ratio, and decreases the two R&D ratio's. The first effect decreases the net profit ratio but the second effect increases this ratio. The derivative of the net profit ratio with respect to entry is therefore ambiguous. We will discuss two cases. Firstly, the traditional or normal case: the first effect dominates the second effect such that entry reduces the net profit ratio. We will first discuss this traditional or normal case, which is illustrated in figure 7.8 (there are two modifications of the base run scenario, a=5 and θ=0.2 in stead of a=2.5 and θ=0.1)[27]. Entry leads eventually to negative profits, so that the number of viable firms in an industry is limited.

The appearance of a negative net profit ratio is also more likely when technological opportunities (θ, θ_1, b, δ, δ_1) are larger. This argument together with the finding that entry traditionally reduces the net profit ratio implies that industries with larger technological opportunities tend to be more concentrated. This relation is also found by Dasgupta and Stiglitz (1980) and is illustrated in figure 7.8. When the own R&D elasticity increases from θ=0.2 to θ=0.22 the number of viable firms in the industry decreases.

A higher degree of Productdifferentation, lower a, causes a decrease in a_p which makes a negative profit ratio less likely. Traditionally, the number of viable firms increases in this market. The model predicts therefore that industries with a higher degree of differentiated goods are normally less concentrated than more homogenous goods, see figure 7.8. When a decreases from a=5 to a=4.8 the number of viable firms increases.

Combining these finding we can conclude that normally (traditionally) industries with high technological opportunities and a low degree of product differentiation are very concentrated and highly differentiated industries with low technological opportunities are characterised by many firms.

We will now discuss the exceptional case, which is illustrated in figure 7.9. Entry first reduces the net profit rate but after a certain level of rivalry it increases this rate. This means that after a certain level of rivalry the influence of entry on the R&D fixed costs ratio's is bigger than the influence on the gross profit ratio (perceived price elasticity). With other words, entry means smaller marketshares but it also reduces own R&D expenditures because entry increases also the knowledge stock, which implies that your own R&D is more productive. The implication of such a pattern is that only a concentrated industry is viable or an industry which is quite competitive (a large number of firms). An intermediate level of rivalry is unviable. Figure 7.9 shows furthermore that industries with higher technological opportunities (higher value of θ) increase the range of unviable market situations which permits only more concentrated industries or more competitive markets. This figure also shows that a higher level of product differentiation (lower value of a) has the opposite results. We can conclude that in comparison with the results in the traditional situation, industries with a lower level of product differentiation or higher technological opportunities are more concentrated when the number of firms is small or characterised by more firms when the number of firms is large.

---

[27]We checked the value of the net-profit ratio for all the other simulations which are described in section 7. In none of the cases was this value negative. The number of firms were therefore not limited in the other simulations. In this case the number of firms is restricted because θ and a are both higher in this situation which both decrease the net profit rate.

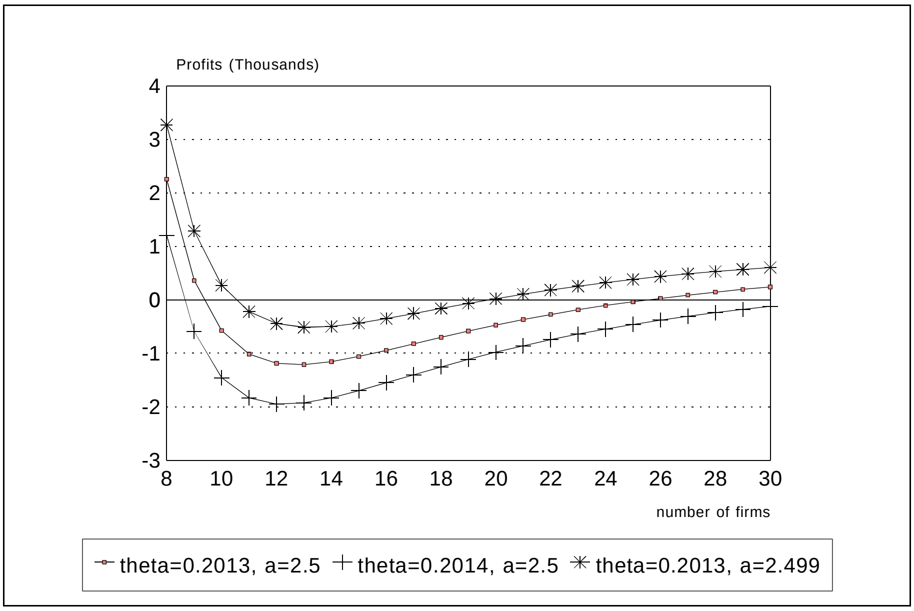

Figure 7.9: An exceptional relation between entry and profits, $m_a=m_q=0.5$, $\varepsilon=1.6$, $b=1.35$, $\xi=1.3$, $\delta=0.1$, $r=0.065$, $\omega_a=\omega_q=0.5$

Welfare

We investigate the influence of entry and the extent of spillovers on welfare. The influence of the extent of process R&D spillovers on welfare is positive. First, a higher extent of process R&D spillovers increases the net profit ratio (see equation (7.14)). Second, according to section 7.3 it also increases the productivity and quality level. This has a positive effect on both the consumer

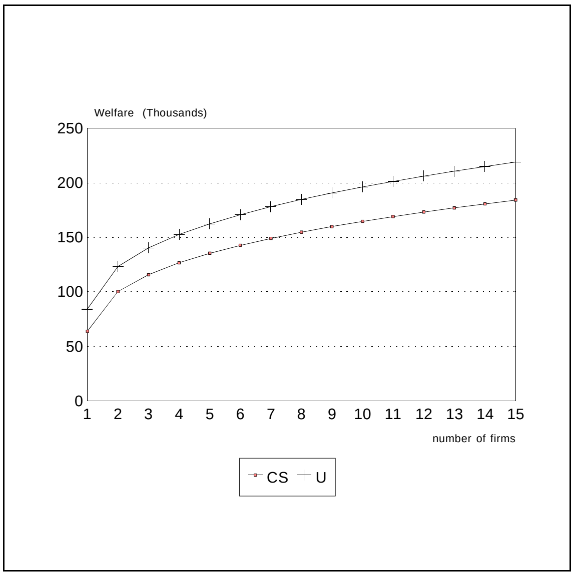

Figure 7.10: The influence of entry on consumer surplus and total welfare given "Love of Variety" (a=2.5).

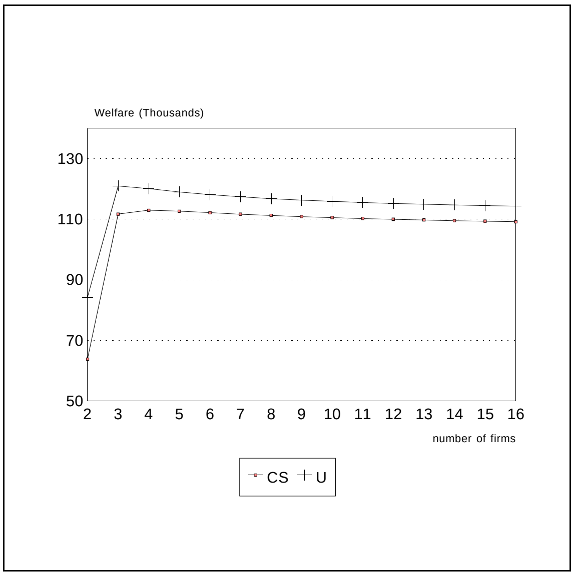

Figure 7.11: The influence of entry on consumer surplus and total welfare given a lower of "Love of variety" (a=7.5)

surplus and the total profit level. Therefore, the extent of process R&D spillovers increases the welfare level.

The influence of entry and the extent of product R&D spillovers on welfare is again ambiguous. We will study the influence of entry. First, entry increases the perceived price elasticity which increases the consumer surplus and decreases or increases the total profits. Second, the influence of entry on the productivity and quality level is ambiguous (see paragraph 7.4). The difference between a and ε and the extent of the spillover effects determine whether the influence is positive or negative. Third, when people possess love of variety, entry increases the consumer surplus directly.

When the products are characterised by love of variety it is typically to find that entry increases welfare. Such a situation is illustrated in figure 7.10. When a firm enters it introduces a new product variety which is appreciated by consumers which exhibit love of variety.

When consumers possess "good characteristic" preferences, large a, the welfare most typically first increases and then decreases with entry. With less differentiated goods, high a, an intermediate level of rivalry stimulates most technological change (see section 7.4). This high level of technological change results in a higher quality and productivity level which is appreciated by consumers and producers. This situation is illustrated in figure 7.11.

If we combine welfare characteristics with the viability of a market, we can distillate an interesting case for government policy. Figure 7.9 showed us that an intermediate level of rivalry is not viable. If we make an additional assumption that the profits have to be zero in the equilibrium only two levels of rivalry satisfy this condition (for the scenario with a=2.5 and θ=2.014 are this 9 and 26 firms). The situation described in figure 7.9 is characterised by love of variety which implies that the welfare level is an increasing function of the number of firms. So if this industry possesses 9 firms the welfare level is lower than in the equilibrium with 26 firms. This industry is "locked in" an inferior industry structure. A temporary government policy, par example a R&D subsidy, is now necessary to bridge the unviable levels of rivalry.

8 Conclusion

This paper treated the complex interplay of spillover effects, productdifferentiation and entry with regard to technological change.

The influence of spillovers on the steady state growth rates were positive. Inter-industry spillover effects had a direct positive influence, whereas the productivity of intra-industry spillovers increased the growth rate multiplier. An interesting result was that the extent of intra-industry spillovers had no influence on the steady state growth rate.

The dynamics of the system were characterised by the convenient property that the system showed saddle point stability when the growth rate multiplier was positive. This implies a unique optimal path for process and product R&D to achieve the steady state.

The steady state R&D intensity was negatively influenced by the extent and positively influenced by the productivity of intra-industry spillovers. Productdifferentiation showed also a negative influence on the R&D intensity. The market structure which yields the highest R&D intensity is dependent on the spillover effects and degree of product differentiation.

The influence of the extent of spillovers on the R&D level can be positive or negative. A positive influence is more likely if leakages are high, the number of firms is higher, products are less differentiated and technological opportunities are larger. But this positive influence of the extend of spillovers on the R&D level is dependent on the "double counting" method of Levin and Reiss. Without this "double counting" method and the in our model introduced threatening spillover pool we obtain the Spence result: a negative influence of the extend of spillover on the R&D level. The introduction of the threatening spillover pool makes a positive and a negative influence of the extend of product R&D spillovers possible on the technological level (i.e. the perceived quality level) possible independent of the "double counting" method.

The technological performance measured by its quality and productivity level is positively dependent on the extend of the productivity spillover. The influence of the extend of the quality spillover and entry on technological performance is again ambiguous. The influence of entry on the productivity level is complicated. A very interesting pattern is that, given a small difference between the elasticity of substitution and the inter-industry price elasticity of demand, entry first decreases and then increases the productivity level.

Normally, negative profits limit the number of firms in an industry. High technological opportunities and lowly differentiated products leads normally to more concentrated products. But in this model it is also possible that high technological opportunities and lowly differentiated products cause an industry structure which is characterised by a large number of firms. Only an intermediate level of rivalry is not viable. Depending on the welfare implication a concentrated or a competitive industry is desirable. Temporary government policy is necessarily if one industry is locked in the wrong industry structure.

Welfare is enhanced by productivity spillover effects. The influence of the extend of quality spillovers is ambiguous. In general we can say that when consumers show love of variety entry increases welfare. But when consumers are only interested in product characteristics an intermediate level of rivalry generates the highest welfare level.

Appendix A: "Derivation of Demand Functions"

Two Stage Utility Maximization subject to a Budget Constraint: "Love of Variety" Approach.

First Stage: Minimisation of expenditures to achieve a certain level of quality characteristics of differentiated goods.

The total quality characteristics index is:

$$D = \left[\sum_{i=1}^{n} (y_i . Q_{p,i}^{b})^{\rho}\right]^{\frac{1}{\rho}} = \left[\sum_{i=1}^{n} c_i . y_i^{\rho}\right]^{\frac{1}{\rho}} \tag{A.1}$$

where $c_i = Q_{p,i}^{b.\rho}$. The quality level of a product is given for a consumer. We limit attention to symmetric equilibria which implies that the quality level of different products will be the same. Consumers minimize expenditures to achieve a certain level of quality characteristics.

$$\underset{y_i,\lambda}{L} = \sum_{i=1}^{n} p_i . y_i - \lambda . \left[[\sum_{i=1}^{n} c_i . y_i^{\rho}]^{\frac{1}{\rho}} - D \right] \tag{A.2}$$

The first order conditions associated with this problem are:

$$\frac{\partial L}{\partial y_i} \Rightarrow \frac{p_i}{\lambda} = c_i . \left(\frac{y_i}{D}\right)^{\rho-1} \tag{A.3}$$

$$\frac{\partial L}{\partial \lambda} \Rightarrow 1 = \sum_{i=1}^{n} c_i . \left(\frac{y_i}{D}\right)^{\rho} \tag{A.4}$$

Taking equation (A.3) to the power $\rho/(\rho-1)$ gives

$$\left(\frac{p_i}{\lambda . c_i}\right)^{\frac{\rho}{\rho-1}} = \left(\frac{y_i}{D}\right)^{\rho} \tag{A.5}$$

Substitution of equation (A.5) in equation (A.4) gives:

$$1 = \sum_{i=1}^{n} c_i . \left(\frac{p_i}{\lambda . c_i}\right)^{\frac{\rho}{\rho-1}} \tag{A.6}$$

Solving equation (A.6) for λ gives

$$\lambda = \left[\sum_{i=1}^{n} c_i^{\frac{1}{1-\rho}} . p_i^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}} \qquad \textbf{(A.7)}$$

Using total expenditures (C) and equation (A.5), it is easy to verify

$$C = \sum_{i=1}^{n} p_i . y_i = \lambda . \sum_{i-1}^{n} y_i . c_i \left(\frac{y_i}{D} \right)^{\rho-1} = \lambda . \sum_{i=1}^{n} [c_i . y_i^{\rho}] . D^{1-\rho} = \lambda . D \qquad \textbf{(A.8)}$$

λ is therefore the priceindex (P_D) for the quality characteristic index (D). Using Shephard's lemma we find

$$\frac{\partial C}{\partial p_i} = y_i = \frac{\partial \lambda}{\partial p_i} . D \qquad \textbf{(A.9)}$$

The demand function conditional on the quality characteristic index can be derived using equation (A.7) and equation (A.9).

$$y_i = \left[\sum_{i=1}^{n} c_i^{\frac{1}{1-\rho}} . p_i^{\frac{\rho}{\rho-1}} \right]^{-\frac{1}{\rho}} . c_i^{\frac{1}{1-\rho}} . p_i^{\frac{1}{\rho-1}} . D = c_i^{\frac{1}{1-\rho}} . \left(\frac{p_i}{P_D} \right)^{\frac{1}{\rho-1}} . D \qquad \textbf{(A.10)}$$

Remember that $P_D = \lambda$.

Second Stage: Choose allocation of expenditures across goods D and H to maximize total utility subject to the overall budget constraint.

The maximisation problem is

$$\underset{D,H,\mu}{L} = \frac{X_0}{\zeta} . D^{\zeta} + H - \mu . (P_D . D + H - I) \qquad \textbf{(A.11)}$$

where I=income in terms of the numeraire, H=homogenous good (numeraire) The first order conditions are

$$\frac{\partial L^{//}}{\partial D} = X_0 . D^{\zeta-1} - \mu . P_D = 0 \qquad \textbf{(A.12)}$$

$$\frac{\partial L^{//}}{\partial H} = 1 - \mu = 0 \qquad \textbf{(A.13)}$$

Combining these first order conditions we derive immediately the level of the quality characteristics dependent on the price index.

$$D = \left(\frac{P_D}{X_0}\right)^{\frac{1}{\zeta-1}} \quad \textbf{(A.14)}$$

The result of the first stage (equation (A.10)) and the result of the second stage (equation (A.14)) together give the demand equation for each differentiated good ($c_i=Q_{pi}^{\rho}$).

$$y_i = X_0^{\varepsilon}.Q_{p,i}^{b.(a-1)}.p_i^{-a}.P_D^{a-\varepsilon} \quad \textbf{(A.15)}$$

where a=1/(1-ρ)>1 and ε=1/(1-ζ)>1.
The definition of P_D becomes now (redefine equation (A.7))

$$P_D = \left[\sum_{i=1}^{n} Q_{p,i}^{b.(a-1)}.p_i^{1-a}\right]^{\frac{1}{1-a}} \quad \textbf{(A.16)}$$

Appendix B: Calculation of Steady State Growth Rates

In this appendix we derive the steady state growth rates for the model in section four.
The current value hamiltonian associated with the maximization in section four is

$$H_c(p_i,\ R_{c,i},\ R_{d,i},\lambda,\mu) = \left[p_i.y_i(p_i,Q_{p,i}) - TC_i(y_i,A_i,Q_{p,i}) - R_{c,i}.q_c - R_{d,i}.q_d\right].dt + \mu.(\eta.A_i^{m_a}.R_{c,i}^{\theta}.T_{c,i}^{\theta_1}.S_{c,i}^{\theta_2}) + \lambda.\left(\frac{\gamma.Q_{p,i}^{m_q}.R_{d,i}^{\delta}.T_{d,i}^{\delta_1}.S_{d,i}^{\delta_2}}{W_{d,i}^{\delta_3}}\right) \quad \textbf{(B.1)}$$

where

$$y_i = X_0^{\varepsilon}.Q_{p,i}^{b.(a-1)}.p_i^{-a}.P_D^{a-\varepsilon} \quad ; \quad P_D = \left[\sum_{i=1}^{n} Q_{p,i}^{b.(a-1)}.p_i^{1-a}\right]^{\frac{1}{1-a}}$$

$$TC_i = \frac{Q_{p,i}^{\xi}}{A_i}.y_i.w^{1-\alpha}.v^{\alpha}.(1-\alpha)^{\alpha-1}.\alpha^{-\alpha} = \frac{Q_{p,i}^{\xi}}{A_i}.y_i.Z_1 \quad ; \quad Z_1 = \left(\frac{w}{1-\alpha}\right)^{1-\alpha}.\left(\frac{v}{\alpha}\right)^{\alpha} \quad \textbf{(B.2)}$$

$$dA_i/dt = \eta.A_i^{m_a}.R_{c,i}^{\theta}.T_{c,i}^{\theta_1}.S_{c,i}^{\theta_2}$$

$$dQ_{p,i}/dt = \left(\frac{\gamma.Q_{p,i}^{m_q}.R_{d,i}^{\delta}.T_{d,i}^{\delta_1}.S_{d,i}^{\delta_2}}{W_{d,i}^{\delta_3}}\right)$$

First order conditions:

$$\frac{\delta H_c}{\delta p_i} = y_i + p_i \frac{\partial y_i}{\partial p_i} - \frac{\partial TC_i}{\partial y_i}.\frac{\partial y_i}{\partial p_i} \Rightarrow p_i = \left(\frac{a_p}{a_p - 1}\right).\frac{TC}{y_i} \quad ; \quad a_p > 1 \tag{B.3}$$

where a_p is the perceived price elasticity. The derivation of this elasticity is

$$a_p = -\frac{\partial y_i}{\partial p_i}.\frac{p_i}{y_i} = a - (a-\varepsilon).\frac{Q_{p,i}^{b.(a-1)}.p_i^{1-a}}{\sum_{i=1}^{n} Q_{p,i}^{b.(a-1)}.p_i^{1-a}}.\frac{n}{n} = a - \frac{(a-\varepsilon)}{n} \tag{B.4}$$

Using the dynamic constraint, and equation (3.3), $T_{c,i} = R_{c,i} + \omega_a \sum_{j \neq i}^{n} R_{c,j}$, and the assumption that firms assume that other firms will not change their R&D strategy: $dT_{c,i}/dR_{c,i}=1$.
The first order condition with respect to R_c becomes

$$\frac{\delta H_c}{\delta R_{c,i}} \Rightarrow q_c = \mu.\left[\frac{\theta.(dA_i/dt)}{R_{c,i}} + \frac{\theta_1.(dA_i/dt)}{T_{c,i}}\right] \tag{B.5}$$

In a symmetric equilibrium are the R&D expenditures equal across firms ($R_{c,i}=R_{c,j}$). Equation (B.5) becomes in the symmetric equilibrium

$$\frac{\delta H_c}{\delta R_{c,i}} \Rightarrow q_c = \mu.\frac{dA_i/dt}{R_{c,i}}.\left[\theta + \frac{\theta_1}{1+\omega_a.(n-1)}\right] \tag{B.6}$$

In the same way we can calculate the derivative with respect to $R_{d,i}$ (use equation (3.8) and equation (3.7))

$$\frac{\delta H_c}{\delta R_{d,i}} \Rightarrow q_d = \lambda.\frac{(dQ_{p,i}/dt)}{R_{d,i}}.\left[\delta + \frac{(\delta_1 - \omega_q.\delta_3)}{1+\omega_q.(n-1)}\right] \tag{B.7}$$

The first order conditions with respect to the state variables $Q_{p,i}$ and A_i are

$$\frac{\delta H_c}{\delta A_i} \Rightarrow -\frac{d\mu}{dt} + \mu.r = \frac{TC_i}{A_i} + \mu.m_a.\left(\frac{\frac{dA_i}{dt}}{A_i}\right) \Rightarrow -\frac{\delta\mu}{\delta t} = \frac{TC_i}{A_i} - \mu.(r - m_a.\hat{A}_i) \tag{B.8}$$

$$\frac{dH}{dQ_{p,i}} \Rightarrow -\frac{d\lambda}{dt}+r.\lambda = -\zeta.\frac{TC_i}{Q_{p,i}}-\frac{dTC_i}{dy_i}.\frac{dy_i}{dQ_{p,i}}+p_i.\frac{dy_i}{dQ_i}+\left(\frac{\frac{dQ_{p,i}}{dt}}{Q_{p,i}}\right).m_q.\lambda \tag{B.9}$$

As in the case with the perceived price elasticity of demand we can calculate the perceived quality elasticity of demand (b_p).

$$b_p = \frac{\partial y_i}{\partial Q_{p,i}}.\frac{Q_{p,i}}{y_i} = b.(a-1)+\frac{(a-\varepsilon)}{1-a}.b.(a-1).\frac{Q_{p,i}^{b.(a-1)}.p_i^{1-a}}{\sum_{i=1}^{n} Q_{p,i}^{b.(a-1)}.p_i^{1-a}}.\frac{n}{n}$$
$$= b.(a-1)-\frac{(a-\varepsilon).b}{n} = b.(a_p-1) \tag{B.10}$$

Combining the last two equation gives

$$\frac{dH_c}{dQ_{p,i}} \Rightarrow -\frac{d\lambda}{dt}+\lambda.(r-m_q.\hat{Q}_{p,i})) = -\zeta.\frac{TC_i}{Q_{p.i}}-\frac{TC_i}{y_i}.b_p.\frac{y_i}{Q_{p,i}}+p_i.b_p.\frac{y_i}{Q_{p,i}}$$
$$= (-b_p-\zeta).\frac{TC_i}{Q_{p,i}}+b_p.\left[\frac{a_p}{a_p-1}\right].\frac{TC_i}{Q_{p,i}} = \left[\frac{b_p-\zeta.(a_p-1)}{a_p-1}\right].\frac{TC_i}{Q_{p,i}} = (b-\zeta).\frac{Tc_i}{Q_{p,i}} \tag{B.11}$$

where use have been made of equation (B.3).
The two Transversality conditions (TVC) are

$$TVC_\lambda = \lim_{t\to\infty} \lambda_t.e^{(\sigma-r).t} = 0$$
$$TVC_\mu = \lim_{t\to\infty} \mu_t.e^{(\sigma-r).t} = 0 \tag{B.12}$$

The dynamic constraints equation (3.2) and equation (3.6) are the other first order conditions.

When we put these first order conditions in growth rates and calculate the steady state growth rates of the redefined variables we find that the steady state growth rates of A_i", $Q_{p,i}$", $R_{d,i}$", $R_{c,i}$", λ and μ are equal to zero. A zero growth rate for λ and μ implies that the transversality condition is satisfied when $r > \sigma$.

$$TVC \Rightarrow r-\sigma > 0 \tag{B.13}$$

Calculation of steady state growth rates in a symmetric equilibrium

The growth rate of the demand function (i.e. equation (2.6)) in a symmetric situation is

$$\hat{y}_i = \varepsilon.\hat{X}_0 + b.(\varepsilon-1).\hat{Q}_{p,i} - \varepsilon.\hat{p}_i \qquad \textbf{(B.14)}$$

The growth rate of the total variable costs (TC) is

$$\hat{TC}_i = \hat{y}_i + \xi.\hat{Q}_{p,i} + \hat{Z}_1 - \hat{A}_i \qquad \textbf{(B.15)}$$

The constant growth rates of the first order conditions ((B.3), (B.6), (B.7), (B.8) and (B.11)) are resp.

$$\hat{p}_i = \hat{TC}_i - \hat{y}_i \qquad \textbf{(B.16)}$$

$$\hat{q}_c = \hat{\mu} + \hat{A}_i - \hat{R}_{c,i} \qquad \textbf{(B.17)}$$

$$\hat{q}_d = \hat{\lambda} + \hat{Q}_{p,i} - \hat{R}_{d,i} \qquad \textbf{(B.18)}$$

$$\hat{\mu} = \hat{TC}_i - \hat{A}_i \qquad \textbf{(B.19)}$$

$$\hat{\lambda} = \hat{TC}_i - \hat{Q}_{p,i} \qquad \textbf{(B.20)}$$

The constant growth rates of the productivity (equation 3.2) and quality generation (equation 3.5) process are respectively:

$$0 = \theta.\hat{R}_{c,i} + \theta_1.\hat{R}_{c,i} + \theta_2.\hat{S}_{c,i} - (m_a - 1).\hat{A}_i \qquad \textbf{(B.21)}$$

$$0 = \delta.\hat{R}_{d,i} + \delta_1.\hat{R}_{d,i} - \delta_3.\hat{R}_{d,i} + \delta_2.\hat{S}_{d,i} - (m_q - 1).\hat{Q}_{p,i} \qquad \textbf{(B.22)}$$

Combining equations (B.17) and (B.19) gives

$$\hat{q}_c = \hat{TC}_i - \hat{R}_{c,i} \qquad \textbf{(B.23)}$$

Combining equations (B.18) and (B.20)

$$\hat{q}_d = \hat{TC}_i - \hat{R}_{d,i} \qquad \textbf{(B.24)}$$

First substitute the growth rate of p_i from equation (B.16) in equation (B.14) and solve this result for $\hat{y}_i$. Substituting this result in equation (B.15) we get

$$\hat{TC}_i = \varepsilon.\hat{X}_0 + (b-\xi).(\varepsilon-1).\hat{Q}_{p,i} + (1-\varepsilon).\hat{Z}_1 + (\varepsilon-1).\hat{A}_i \qquad \textbf{(B.25)}$$

Substitute (B.25) in respectively equation (B.23) and equation (B.24). Solve one of the two resulting equations for $\hat{R}_{d,i}$ and substitute the result in the other equation. The steady state growth rate for $R_{c,i}$ is

$$\hat{R}^*_{c,i} = \frac{\varepsilon.\hat{X}_0 \ -\hat{q}_c + \ (\varepsilon-1).\left[-\hat{Z}_1+(b-\xi).\left(\frac{\delta_2}{1-m_q}.\hat{S}_{d,i}-\frac{\delta+\delta_1-\delta_3}{1-m_q}.(\hat{q}_d-\hat{q}_c)\right)+\frac{\theta_2}{1-m_a}.\hat{S}_{c,i}\right]}{1-(\varepsilon-1).(b-\xi).\left(\frac{\delta+\delta_1-\delta_3}{1-m_q}\right)-(\varepsilon-1).\left(\frac{\theta+\theta_1}{1-m_a}\right)} \quad \textbf{(B.26)}$$

Using equation (B.21) and equation (B.26) we can derive the steady state growth rate for the productivity level (A_i)

$$\hat{A}^*_i \ = \ \left(\frac{\theta+\theta_1}{1-m_a}\right).\hat{R}^*_{c,i}+\left(\frac{\theta_2}{1-m_a}\right).\hat{S}_{c,i} \quad \textbf{(B.27)}$$

Equation (B.23) and equation (B.24) together show that the steady state growth rates of the product and process R&D budget are equal. The steady state growth rate for product R&D is then

$$\hat{R}^*_{d,i} \ = \ \hat{q}_c \ + \ \hat{R}^*_{c,i}-\hat{q}_d \quad \textbf{(B.28)}$$

Equation (B.22) gives the steady state growth rate for the quality level ($Q_{p,i}$)

$$\hat{Q}^*_{p,i} \ = \ \left(\frac{\delta+\delta_1-\delta_3}{1-m_q}\right).\hat{R}^*_{d,i}+\left(\frac{\delta_2}{1-m_q}\right).\hat{S}_{d,i} \quad \textbf{(B.29)}$$

Appendix C: Time Elimination[28]

As in Van Meijl and Van Zon (1993) we deflate all variables with their steady state growth rates. Two examples are

$$\begin{aligned} A^{//}_i \ &= \ A_i.e^{-\sigma_a.t} \\ Q^{//}_{p,i} \ &= \ Q_{p,t}.e^{-\sigma_q.t} \end{aligned} \quad \textbf{(C.1)}$$

where σ_q and σ_a are respectively the steady state growth rates of $Q_{p,i}$ and A_i. We defined $R_{c,i}$", $R_{d,i}$", y_i" and p_i" in the same way.
The two dynamic constraints with the deflated variables become

[28]For an elaborated treatment of the time elimination method see, Van Meijl and Van Zon (1993).

$$\frac{dA_i^{//}}{dt} = \eta . A_i^{//\ m_a} . R_{c,i}^{//\ \theta} . T_{c,i}^{//\ \theta_1} . S_c^{//\ \theta_2} - \sigma_a . A_i^{//} \quad \textbf{(C.2)}$$

$$\frac{dQ_{p,i}^{//}}{dt} = \frac{\gamma . Q_{p,i}^{//\ m_q} . R_{d,i}^{//\ \delta} . T_{d,i}^{//\ \delta_1} . S_d^{//\ \delta_2}}{W_{d,i}^{//\ \delta_3}} - \sigma_q . Q_{p,i}^{//} \quad \textbf{(C.3)}$$

The Hamiltonian of the profit maximization problem with the redefined variables is

$$H_c(p_i^{//}, R_{c,i}^{//}, R_{d,i}^{//}, \lambda, \mu) =$$
$$\int_0^{\infty} e^{(\sigma - r).t} \left[p_i^{//} . y_i^{//} (p_i^{//}, Q_{p,i}^{//}) - TC_i^{//} (y_i^{//}, A_i^{//}, Q_{p,i}^{//}) - R_{c,i}^{//} . q_{c,0} - R_{d,i}^{//} . q_{d,0} \right] dt$$
$$+ \mu^{//} . (\eta . A_i^{//\ m_a} . R_{c,i}^{//\ \theta} . T_{c,i}^{//\ \theta_1} . S_{c,i}^{//\ \theta_2} - \sigma_a . A_i^{//}) \quad \textbf{(C.4)}$$
$$+ \lambda^{//} . \left(\frac{\gamma . Q_{p,i}^{//\ m_q} . R_{d,i}^{//\ \delta} . T_{d,i}^{//\ \delta_1} . S_{d,i}^{//\ \delta_2}}{W_{d,i}^{\delta_3}} - \sigma_q . Q_{p,i}^{//} \right)$$

$$y_i^{//} = X_{0,0}^{\varepsilon} . Q_{p,i}^{//\ b.(a-1)} . p_i^{//\ -a} . P_D^{//\ a-\varepsilon}$$
$$P_D^{//} = \left[\sum_{i=1}^{n} Q_{p,i}^{//\ b.(a-1)} . p_i^{//\ 1-a} \right]^{\frac{1}{1-a}} \quad \textbf{(C.5)}$$
$$TC_i^{//} = \frac{Q_{p,i}^{//\ \xi}}{A_i^{//}} . y_i^{//} . w_0^{1-\alpha} . v_0^{\alpha} . (1-\alpha)^{\alpha-1} . \alpha^{-\alpha} = \frac{Q_{p,i}^{//\ \xi}}{A_i^{//}} . y_i^{//} . Z_0$$

where σ is the steady state growth rate of revenues, R&D budgets and profits and $q_{c,0}$, $q_{d,0}$, $X_{0,0}$, w_0 and v_0 are the initial values of resp. the price of process R&D, the price of product R&D, the exogenous scale of demand, wages and unit cost of capital.
The first order conditions are

$$\frac{\delta H_c}{\delta p_i} \Rightarrow p_i^{//} = \frac{a_p}{a_p - 1} . \frac{TC^{//}}{y_i^{//}} \quad ; \quad a_p > 1 \quad \textbf{(C.6)}$$

The perceived price elasticity (a_p) is give by equation (B.4).

$$\frac{\delta H_c}{\delta R_{c,i}^{//}} \Rightarrow q_{c,0} = \mu^{//} . \frac{(dA_i^{//} / dt + \sigma_a . A_i^{//})}{R_{c,i}^{//}} . \left[\theta + \frac{\theta_1}{1 + \omega_a . (n-1)} \right] \quad \textbf{(C.7)}$$

$$\frac{\delta H_c}{\delta R''_{d,i}} \Rightarrow q_{d,0} = \lambda'' . \frac{(dQ''_{p,i}/dt + \sigma_q . Q''_{p,i})}{R''_{d,i}} . \left[\delta + \frac{\delta_1 - \omega_q . \delta_3}{1 + \omega_q . (n-1)} \right] \quad \textbf{(C.8)}$$

$$\frac{\delta H_c}{\delta A''_i} \Rightarrow -\frac{d\mu''}{dt} + \mu'' . (r - \sigma + (1 - m_a) . \sigma_a - m_a . \hat{A}''_i) = \frac{TC''_i}{A''_i} \quad \textbf{(C.9)}$$

$$\frac{\delta H_c}{\delta Q''_{p,i}} \Rightarrow -\frac{d\lambda''}{dt} + \lambda'' . (r - \sigma + (1 - m_q) . \sigma_q - m_q . \hat{Q}''_{p,i}) = \frac{TC_i}{Q_{p,i}} . (b - \xi) \quad \textbf{(C.10)}$$

Following appendix B it is easy to verify that the steady state growth rates of the redefined system are zero.

Derivation of the four differential equations (symmetric equilibrium)

The dA_i''/dt and $dQ_{p,i}''/dt$ differential equations are given by respectively equation (C.2) and (C.3). To derive the $dR_{c,i}''/dt$ differential equation we first solve equation (C.7) for μ":

$$\mu'' = \frac{q_{c,0} . R''^{\,1-\theta-\theta_1}_{c,i} . A''^{\,-m_a}_i}{\eta . S_{c,0}^{\theta_2} . toa} \quad where\ toa = (1 + \omega_a * (n-1))^{\theta_1} . \left[\theta + \frac{\theta_1}{1 + \omega_a * (n-1)} \right] \quad \textbf{(C.11)}$$

When we differentiate μ" with respect to time we get

$$\frac{\partial \mu''}{\partial t} = \frac{q_{c,0} . A''^{\,-m_a}_i . (R''^{\,-\theta-\theta_1}_{c,i} . (1 - \theta - \theta_1) . \frac{dR_{c,i}}{dt} - m_a . R''^{\,1-\theta-\theta_1}_{c,i} . \hat{A}''_i)}{\eta . S_{c,0}^{\theta_2} . toa} \quad \textbf{(C.12)}$$

When we substitute μ" and dμ"/dt in equation (C.9) we get the $dR_{c,i}''/dt$ differential equation

$$\frac{q_{c,0} . (r - \sigma + (1 - m_a) . \sigma_a)}{\eta . toa . S_{c,0}^{\theta_2}} . (R''_c)^{1-\theta-\theta_1} - \frac{q_{c,0} . (1 - \theta - \theta_1) . (R''_c)^{-\theta-\theta_1}}{\eta . toa . S_{c,0}^{\theta_2}} . \frac{dR''_{c,i}}{dt} =$$

$$X_{0,0}^{\varepsilon} . Z_1^{1-\varepsilon} . \left(\frac{a_p - 1}{a_p} \right)^{\varepsilon} . n^{\frac{a-\varepsilon}{1-a}} (A''_i)^{\varepsilon + m_a - 2} . (Q''_{p,i})^{(b-\xi).(\varepsilon-1)} \quad \textbf{(C.13)}$$

we can simplify this equation to

$$\frac{dR_c^{//}}{dt} = \phi_4.(R_c^{//})^{\theta+\theta_1}.\left(\phi_5.(R_c^{//})^{1-\theta-\theta_1} - \phi_6.(Q_p^{//})^{(b-\xi).(\varepsilon-1)}.(A^{//})^{\varepsilon+m_a-2}\right) \tag{C.14}$$

where
$\phi_4=(\eta.toa.S_{c,0}^{\theta_2})/(q_{c,0}.(1-\theta-\theta_1))$
$\phi_5=(q_{c,0}.(r-\sigma-\sigma_a.(1-m_a))/(\eta.toa.S_{c,0}^{\theta_2})$
$\phi_6=X_{0,0}{}^{\varepsilon}.Z_1{}^{1-\varepsilon}.(1-1/a_p)^{\varepsilon}.n^{(a-\varepsilon)/(1-a)}$

In the same manner we can derive the $dR_{d,i}''/dt$ differential equation. Solve equation (C.8) for λ" and take the time derivative of this equation. Substituting λ" and dλ"/dt in equation (C.10) gives

$$\frac{q_{d,0}.(r-\sigma+(1-m_q).\sigma_q)}{\gamma.doq.S_{d,0}^{\delta_2}}.(R_d^{//})^{1-\delta-\delta_1+\delta_3} - \frac{q_{d,0}.(1-\delta-\delta_1+\delta_3).(R_d^{//})^{-\delta-\delta_1+\delta_3}}{\gamma.doq.S_{d,0}^{\delta_2}}.\frac{dR_{d,i}^{//}}{dt} = \tag{C.15}$$
$$X_{0,0}^{\varepsilon}.Z_1^{1-\varepsilon}.\left(\frac{a_p-1}{a_p}\right)^{\varepsilon}.n^{\frac{a-\varepsilon}{1-a}}.(b-\xi).(A_i^{//})^{\varepsilon-1}.(Q_{p,i}^{//})^{-(1-m-(b-\xi).(\varepsilon-1))}$$

where

$$doq = \frac{(1+\omega_q.(n-1))^{\delta_1-\delta_3}}{(n-1)^{\delta_3}}.\left[\delta+\frac{\delta_1-\omega_q.\delta_3}{1+\omega_q.(n-1)}\right] \tag{C.16}$$

We can simplify equation (C.17) to

$$\frac{dR_d^{//}}{dt} = \phi_1.(R_d^{//})^{\delta+\delta_1-\delta_3}.\left(\phi_2.(R_d^{//})^{1-\delta-\delta_1+\delta_3} - \phi_3.\frac{(A^{//})^{\varepsilon-1}}{(Q_{p,i}^{//})^{1-m_q-(b-\xi).(\varepsilon-1)}}\right) \tag{C.17}$$

where
$\phi_1=(\gamma.doq.S_{d,0}^{\delta_2})/(q_{d,0}.(1-\delta-\delta_1+\delta_3))$
$\phi_2=(q_{d,0}.(r-\sigma-\sigma_q.(1-m_q))/(\gamma.doq.S_{d,0}^{\delta_2})$
$\phi_3=X_{0.0}{}^{\varepsilon}.Z_1{}^{1-\varepsilon}.(1-1/a_p)^{\varepsilon}.n^{(a-\varepsilon)/(1-a)}.(b-\xi)$

In the steady state are dA_i''/dt, $dQ_{p,i}''/dt$, $dR_{c,i}''/dt$ and $dR_{d,i}''/dt$ equal to zero. The steady state level of process R&D is

$$R_c^{//\ *} = \left[\frac{XZN^{m_a'.m_q'}.ESA^{m_q'.(\varepsilon-1)}.\sigma_a^{m_q'.(2-m_a-\varepsilon)-m_a.bed}.ESQ^{m_a'.(\varepsilon-1).(b-\xi)}}{RSA^{m_a'.(m_q'-bed)}.RSQ^{m_a'.bed}.\sigma_q^{ma'.(1-\delta-\delta_1+\delta_3).(\varepsilon-1).(b-\xi)}}\right]^{\frac{1}{m_a'.m_q'-m_a'.bed-m_q'.et}} \tag{C.18}$$

where $m_a'=1-m_a$, $m_q'=1-m_q$, $bed=(b-\xi).(\varepsilon-1)(\delta+\delta_1-\delta_3)$ and $et=(\varepsilon-1).(\theta+\theta_1)$

$$XZN = X_{0,0}^{\varepsilon}.Z^{\varepsilon-1}.(1-1/a_p)^{\varepsilon}.n^{\frac{a-\varepsilon}{1-a}} \quad ; \quad ESA = \eta.S_{c,0}^{\prime\prime\ \theta_2}.(1+\omega_a.(n-1))^{\theta_1} \qquad \textbf{(C.19)}$$

$$RSA = \frac{(r-\sigma+(1-m_a).\sigma_a).q_{c,0}}{\left[\theta+\dfrac{\theta_1}{1+\omega_a.(n-1)}\right]} \quad ; \quad RSQ = \frac{(r-\sigma+(1-m_q).\sigma_q).q_{d,0}}{\left[\delta+\dfrac{\delta_1-\omega_q.\delta_1}{1+\omega_q.(n-1)}\right]}.(b-\xi) \qquad \textbf{(C.20)}$$

$$ESQ = \gamma.S_{d,0}^{\prime\prime\ \delta_2}.(1+\omega_q.(n-1))^{\delta_1-\delta_3}/(n-1)^{\delta_3} \qquad \textbf{(C.21)}$$

The steady state level of product R&D is

$$R_d^{\prime\prime\ *} = \left[\frac{ESA^{m_q^{\prime}.(\varepsilon-1)}.\sigma_q^{m_q^{\prime}.m_a^{\prime}-m_a^{\prime}.(\varepsilon-1).(b-\xi)-m_a^{\prime}.et}.GSQ^{m_a^{\prime}.(\varepsilon-1).(b-\xi)}}{XZN^{-m_a^{\prime}.m_q^{\prime}}.RSA^{m_q^{\prime}.et}.RSQ^{m_q^{\prime}.(m_a^{\prime}-et)}.\sigma_a^{m_q^{\prime}.(1-\theta-\theta_1).(\varepsilon-1)}}\right]^{\frac{1}{m_a^{\prime}.m_q^{\prime}-m_a^{\prime}.bed-m_q^{\prime}.et}} \qquad \textbf{(C.22)}$$

The steady state productivity level can be obtained by substituting the steady state process R&D level in equation C.2:

$$A_i^{\prime\prime\ *} = \left[\frac{\eta.S_{c,0}^{\theta_2}.(1+\omega_a.(n-1))^{\theta_1}}{\sigma_a}\right]^{\frac{1}{1-m_a}}.R_{c,i}^{\prime\prime\ *\frac{\theta+\theta_1}{1-m_a}} \qquad \textbf{(C.23)}$$

The steady state product R&D level and equation (C.4) provide the steady state quality level

$$Q_{p,i}^{\prime\prime\ *} = \left[\frac{\gamma.S_{d,0}^{\delta_2}.(1+\omega_q.(n-1))^{\delta_1-\delta_3}}{\sigma_q.(n-1)^{\delta_3}}\right]^{\frac{1}{1-m_q}}.R_{d,i}^{\prime\prime\ *\frac{\delta+\delta_1-\delta_3}{1-m_q}} \qquad \textbf{(C.24)}$$

Steady State R&D Intensities:

To calculate the steady state R&D intensities we use the first order conditions. First, we solve equation (C.6) for TC. Secondly, solve equation (C.2) for $dA_i''/dt+\sigma_a.A_i''$. Thirdly, solve equation (C.7) for µ". Fourthly, note that dµ"/dt=0 in the steady state. Fifthly, substitute the results of step one to four in equation (C.10). The steady state product R&D intensity is

$$\phi_d = \frac{R_d^{\prime\prime *}.q_{d,0}}{y^{\prime\prime *}.p^{\prime\prime *}} = (b-\xi).\left(1-\frac{1}{a_p}\right).\left(\delta+\frac{\delta_1-\omega_q.\delta_3}{1+\omega_q.(n-1)}\right).\left(\frac{\sigma_q}{r-\sigma+(1-m_q).\sigma_q}\right) \quad \textbf{(C.25)}$$

Using the same five step procedure we can derive the steady state process intensity. In the second step we solve equation (C.3) for $dQ_{p.i}''/dt+(w_q+\sigma_q).Q_{p,i}''$. In the third step we solve equation (C.8) for λ''. The fifth step is substituting all results in equation (C.9).

$$\phi_c = \frac{R_c^{\prime\prime *}.q_{c,0}}{y^{\prime\prime *}.p^{\prime\prime *}} = \left(1-\frac{1}{a_p}\right).\left[\theta+\frac{\theta_1}{1+\omega_a.(n-1)}\right].\left(\frac{\sigma_a}{r-\sigma+(1-m_a).\sigma_a}\right) \quad \textbf{(C.26)}$$

Appendix D: Checking Saddle Point Stability

Feichtinger and Hartl state that the general condition for a saddle point plane without loops is characterised by the following conditions[29]:

$$1)\ \det J > 0 \qquad 2)\ K < 0 \qquad 3)\ 0 < \det J \le K^2/4 \quad \textbf{(D.1)}$$

First, we have to compute the determinant of the Jacobian matrix evaluated at the steady-state point (E):

$$J_E = \begin{bmatrix} -(1-m_q).\sigma_q & 0 & \dfrac{(\delta+\delta_1-\delta_3).\sigma_q}{(Q_{p,i})^{-1}.R_d^{\prime\prime}} & 0 \\ 0 & -(1-m_a).\sigma_a & 0 & \dfrac{(\theta+\theta_1).\sigma_a}{(A_i^{\prime\prime})^{-1}.R_c^{\prime\prime}} \\ \dfrac{-\phi_1.\phi_3.(1-m_q-(b-\xi).(\varepsilon-1))}{R_d^{\prime\prime\ -\delta-\delta_1+\delta_3}.Q_p^{\prime\prime\ 2-m_q-(b-\xi).(\varepsilon-1)}.A^{a-1}} & \dfrac{(1-\varepsilon)\phi_1.\phi_3.R_d^{\prime\prime\ \delta+\delta_1}}{Q_p^{\prime\prime\ 1-m_q-(b-\xi).(\varepsilon-1)}.A^{\prime\prime\ 2-\varepsilon}} & \phi_1.\phi_2.(1-\delta-\delta_1+\delta_3) & 0 \\ \dfrac{-(b-\xi).(\varepsilon-1).\phi_4.\phi_6}{R_c^{\prime\prime\ \theta+\theta_1}.Q_p^{\prime\prime\ 1-(b-\xi).(\varepsilon-1)}.A^{\prime\prime\ 2-\varepsilon-m_a}} & \dfrac{(2-\varepsilon-m_a).\phi_4.\phi_6.R_c^{\prime\prime\ \theta+\theta_1}}{Q_p^{-(b-\xi).(\varepsilon-1)}.A^{\prime\prime\ 3-\varepsilon-m_a}} & 0 & (1-\theta-\theta_1).\phi_4.\phi_5 \end{bmatrix}$$

The determinant of the Jacobian matrix can be simplified to:

[29] For details on the stability of a non-linear differential system with more than one state variable see Feichtinger and Hartl, 1986, p. 122-154.

$$|J| = \phi_1.\phi_4.\phi_2.\phi_5.\sigma_a.(1-m_a).\sigma_q.(1-m_q).\left[1-(b-\xi).(\varepsilon-1).\left(\frac{\delta+\delta_1-\delta_3}{1-m_q}\right)-(\varepsilon-1).\left(\frac{\theta+\theta_1}{1-m_a}\right)\right]$$

The value of the determinant is positive when

$$1>(b-\xi).(\varepsilon-1).\left(\frac{\delta+\delta_1-\delta_3}{1-m_q}\right)+(\varepsilon-1).\left(\frac{\theta+\theta_1}{1-m_a}\right) \qquad \textbf{(D.1)}$$

Therefore when this condition holds the first condition, a positive determinant for the Jacobian matrix, is met.

Second, we have to calculate the value of the following matrix K:

$$K = \begin{bmatrix} \frac{\delta\dot{Q}_p^{//}}{\delta Q_p^{//}} & \frac{\delta\dot{Q}_p^{//}}{\delta R_d^{//}} \\ \frac{\delta\dot{R}_d^{//}}{\delta Q_p^{//}} & \frac{\delta\dot{R}_d^{//}}{\delta R_d^{//}} \end{bmatrix} + \begin{bmatrix} \frac{\delta\dot{A}^{//}}{\delta A^{//}} & \frac{\delta\dot{A}^{//}}{\delta R_c^{//}} \\ \frac{\delta\dot{R}_c^{//}}{\delta A^{//}} & \frac{\delta\dot{R}_c^{//}}{\delta R_c^{//}} \end{bmatrix} + 2.\begin{bmatrix} \frac{\delta\dot{Q}_p^{//}}{\delta A^{//}} & \frac{\delta\dot{Q}_p^{//}}{\delta R_c^{//}} \\ \frac{\delta\dot{R}_d^{//}}{\delta A^{//}} & \frac{\delta\dot{R}_d^{//}}{\delta R_c^{//}} \end{bmatrix}$$

The value of variable K in this model is

$$K = -\phi_1.\phi_2.\omega_q.\left[1-m_q-(b-\xi).(\varepsilon-1).(\delta+\delta_1-\delta_3)\right]-\phi_4.\phi_5.\sigma_a.\left[1-m_a-(\varepsilon-1).(\theta+\theta_1)\right]$$

The second condition, a negative value for K, is satisfied when the condition described in equation (D.1) holds. The third condition, $0<\det J\leq K^2/4$, is also satisfied if the first condition holds. We can conclude that this system is saddle point stable when the condition described in equation (D.1) holds.

Appendix E: Values of Parameters and Exogenous Variables used in Base Run Scenario

a	2.5		S_c	2
ε	1.5		S_d	2
δ	0.15		w	0.8
δ_1	0.075		v	0.6
δ_2	0.05		X_0	100
δ_3	0.05		q_c	0.8
θ	0.1		q_d	0.8
θ_1	0.075		r	0.07
θ_2	0.05		ω_a	0.3
b	1.4		ω_q	0.3
ν	1.1		$\hat{X}_0$	0.04
m_a	0.05		$\hat{w}$	0.02
m_q	0.05		$\hat{v}$	0.02
τ	1		$\hat{q}_c$	0.02
η	1		$\hat{q}_d$	0.02
α	0.25		$\hat{S}_d$	0.02
			$\hat{S}_c$	0.02

References

- Aghion, P. and Howitt, (1990), "A Model of Growth Through Creative Destruction", *NBER Working Paper*, 3223.
- Chiang, A.C. (1984), *"Fundamental Methods of Mathematical Economics"*, Third Edition, McGraw-Hill Book Company, Singapore.
- Cohen, W.M. and D.A. Levinthal (1989), "Innovation and learning: two faces of R&D", *The Economic Journal*, 99, pp.569-596.
- Dasgupta, P. and J.E. Stiglitz (1980), "Industrial structure and the nature of innovative activity", *Economic Journal*, 90, pp. 266-293.
- De Bondt, R. and P. Slaets and B. Cassiman (1992), "The degree of spillovers and the number of rivals for maximum effective R&D, *International Journal of Industrial Organisation*, 10, pp. 35-54.
- Delbono, F. and V. Denicolo (1990), "R&D investment in a symmetric and homogenous oligopoly: Bertrand vs. Cournot", *International Journal of Industrial Organisation*, 8, pp. 297-314.
- Dixit, A.K. and Stiglitz, J.E. (1977), "Monopolistic competition and optimum product diversity", *American Economic Review*, Vol. 67, pp. 297-308.
- Dorfman, R. and Steiner, P.O. (1954), "Optimal Advertising and Optimal Quality", *American Economic Review*, Vol. 44, pp. 826-836.
- Feichtinger G. and R.F. Hartl (1986), *"Optimale Kontrolle Okonomischer Processe: Anwendungen des Maximum prinzips in den Wirtschaftwissenschaften"*, de Gruyter, Berlin.
- Grossman, G.M. and E. Helpman (1991a), "Quality ladders in the Theory of Growth", *Review of Economic Studies*, vol. 58, pp. 86-91.
- Grossman, G.M. and E. Helpman (1991b), *"Innovation and Growth in the Global Economy"*, Cambridge Mass.:MIT Press.
- Gould, J.P. (1970), "Diffusion Processes and Optimal Advertising Policy", in E.S. Phelps (ed.), *Microeconomic Foundations of Employment and Inflation Theory*, New York.
- Helpman, E. and P.R. Krugman (1989), *"Trade policy and market structure"*, MIT Press, Cambridge.
- Lee, T. and L.L. Wilde (1980), Market structure and innovation, Quarterly Journal of Economics, 93, pp. 395-410.
- Levin, R.C. and P.C. Reiss (1988),"Cost-reducing and Demand-Creating R&D with Spillovers", *RAND Journal of Economics*, vol.19, No. 4, Winter.
- Loury, G.C. (1979), "Market structure and innovation: A reformulation", *Quarterly Journal Economics*, 93, pp. 395-410.
- Lucas, R.E.B. (1988), "On the mechanics of Economic Development", *Journal of Monetary Economics*, vol 22,1 pp. 3-42.
- Meijl, H. van and A.H. van Zon (1993), "Endogenous technological change by cost-reducing and demand-creating Innovations", *MERIT Research memorandum* 93-027, Maastricht.
- Nerlove, M. and K.J. Arrow (1962), "Optimal Advertising Policy under Dynamic Conditions", *Economica*, pp. 124-142.
- Romer, P.M. (1986), "Increasing returns and Long Run Growth", *Journal of political Economy*, vol 94, 5, pp. 1002-1037.
- Romer, P.M. (1990), "Endogenous Technological Change and Growth",

Journal of political Economy, vol 98, no. 2, pp. S71-S102.

- Sato, R. and G.S. Suzawa (1982), *"Research and Productivity, Endogenous Technological Change, Auburn House Publishing Company"*, Boston.
- Spence, A.M. (1976), "Product selection, fixed costs, and monopolistic competition", *Review of Economic Studies*, 43, pp. 217-235.
- Spence, A.M. (1984), "Cost Reduction, Competition and Industry Performance", *Econometrica*, 52, pp.101-121.